In this monograph we present a novel approach to the problems raised by higher complexity in both nature and the human society, by considering the most complex levels of objective existence as ontological meta-levels, such as those present in the creative human minds and civilised, modern societies. Thus, a 'theory' about theories is called a *'meta-theory'*. In the same sense that a statement about propositions is a higher-level $\langle$*proposition*$\rangle$ rather than a simple proposition, a global process of subprocesses is a *meta-process*, and the emergence of higher levels of reality *via* such meta-processes results in the objective existence of *ontological meta-levels.* The new concepts suggested for understanding the emergence and evolution of life, as well as human consciousness, are in terms of globalisation of multiple, underlying processes into the meta-levels of their existence. Such concepts are also useful in computer aided ontology and computer science [1],[194],[197]. The selected approach for our broad– but also in-depth– study of the fundamental, relational structures and functions present in living, higher organisms and of the extremely complex processes and meta-processes of the human mind combines new concepts from three recently developed, related mathematical fields: Algebraic Topology (AT), Category Theory (CT) and Higher Dimensional Algebra (HDA), as well as concepts from multi-valued logics. Several important relational structures present in organisms and the human mind are naturally represented in terms of universal CT concepts, variable topology, non-Abelian categories and HDA-based notions. The unifying theme of local-to-global approaches to organismal development, biological evolution and human consciousness leads to novel patterns of relations that emerge in super- and ultra-complex systems in terms of global compositions of local procedures [33],[39]. This novel AT concept of *combination of local procedures* is suggested to be relevant to both ontogenetic development and organismal evolution, beginning with the origin of species of higher organisms; such concepts may provide a formal framework for an improved understanding of evolutionary biology and the origin of species on multiple levels–from molecular to species and biosphere levels. It is claimed that human consciousness is an *unique* phenomenon which should be regarded as a composition, or combination of ultra-complex, global processes of subprocesses, at a *meta-level* supported by, and compatible with, the human brain dynamics [11]–[23],[33]. Thus, a defining characteristic of such conscious processes involves a *combination of global procedures* or meta-processes– that may also involve parallel processing of

both image and sound sensations, perceptions, emotions and decision making, etc.– that ultimately leads to the ontological meta-level of the ultra-complex, human mind. Then, an extension of the concept of co–evolution of human consciousness and society leads one to the concept of *social consciousness* [190]. One arrives also at the conclusion that the human mind and consciousness are the result not only of the *co-evolution* of man and his society [91],[186],[190], but that they are, in fact, the result of the original *co-emergence* of the meta-level of a minimally-organized human society with that of several, ultra-complex human brains. The human 'spirit' and society are, thus, *completely inseparable*–just like the very rare Siamese twins. Therefore, the appearance of human consciousness is considered to be critically dependent upon the societal co-evolution, the emergence of an elaborate language-symbolic communication system, as well as the existence of 'virtual', higher dimensional, non–commutative processes that involve separate space and time perceptions in the human mind. Two fundamental, logic adjointness theorems are considered that provide a logical basis for categorical representations of functional genome and organismal networks in variable categories and extended toposes, or topoi, 'classified' (or encoded) by multi-valued logic algebras; their subtly nuanced connections to the variable topology and multiple geometric structures of developing organisms are also pointed out. Our ultra-complexity viewpoint throws new light on previous semantic models in cognitive science and on the theory of levels formulated within the framework of Categorical Ontology [40],[69]. A paradigm shift towards *non-commutative*, or more generally, non-Abelian theories of highly complex dynamics [33],[40],[69] is suggested to unfold now in physics, mathematics, life and cognitive sciences, thus leading to the realizations of higher dimensional algebras in neurosciences and psychology, as well as in human genomics, bioinformatics and interactomics. The presence of strange attractors in modern society dynamics, and especially the emergence of new meta-levels of still-higher complexity in modern society, gives rise to very serious concerns for the future of mankind and the continued persistence of a multi-stable Biosphere if such ultra-complexity, meta-level issues continue to be ignored by decision makers.

KEYWORDS: *Categorical Ontology and the Theory of Levels (COTL); meta-levels; Non-Abelian Categorical Ontology; analysis and synthesis; Theoretical Biology; General Systems Theory and Complex Systems Biology; closed and open systems; boundaries and horizons; complex, super–complex and ultra–complex system dynamics; nonlinear dynamics; Autopoiesis and generalised metabolic–replication systems; (M,R)-systems (MRs) and organisms; Theory of Categories, Functors and Natural Transformations (CT); Yoneda–Grothendieck Lemma; category of categories, super-category, or 2-category; n-category; ETAC and ETAS axioms; Non–Abelian Algebraic Topology (NAAT); Double Groupoids, category of double groupoids and double category; Higher Homotopy–Generalised van Kampen theorems (HHGvKTs); Higher Dimensional Algebra (HDA) of Networks; Higher Dimensional Algebra of Brain Functions; non–commutative topological invariants of complex dynamic state spaces; Quantum Algebraic Topology (QAT) and Axiomatic Quantum Theory (AQT); Quantum Double Groupoids; artificial intelligence (AI) and Biomimetics; automata vs. quantum automata and organisms; Łukasiewicz-Moisil (LM) Logic Algebras of Genetic Networks and Interactomes; LM- and Q- Logic; Relational Biology Principles; Organismic Supercategories (OS) and Categories of Relational Patterns; Natural Transformations in Molecular and Relational Biology; molecular class variables (mcv); Similarity, Analogous*

and Adjoint Systems as adjoint functors; weak adjointness, nuclear transplants and cloning; the origin of life and primordial MR models; universal properties in CT; pushouts, pullbacks, cones and co-cones; duality; categorical limits, colimits and chains of local procedures in developmental and evolutionary biology; biogroupoids, variable groupoids, variable categories, locally Lie groupoids and groupoid atlas structures; local-to-global aspects of Biological Evolution; Compositions of Local Procedures (COLPs); co-evolution of human society and the human mind; Human Consciousness and Synaesthesia; Co-emergence of human consciousness and society; variable biogroupoids, variable biotopology and variable categories; Rosetta biogroupoids of human social interactions; social interactions, objectivation and memes; anticipation; strange attractors of modern society dynamics

2000 *Mathematics Subject Classification*: 16B50, 68Q15,03G20 (Algebraic logic:: Łukasiewicz algebras), 03G12 (Quantum logic), 93A30 (Systems theory; control :: General :: Mathematical modeling), 18A15 (Category theory; General theory of categories and functors : Foundations, relations to logic), 18A40 (Category theory: General theory of categories and functors:: Adjoint functors), 93B15 (Systems theory; control:: Realizations from input-output data), 92D10 (Genetics and population dynamics::Genetics), 18B40 (Category theory: Special categories :: Groupoids, semigroupoids, semigroups, groups), 18G55 (Category theory;Homotopical algebra), 55U40 (Algebraic topology: Applied category theory:: Foundations of homotopy theory).

1. Introduction

Ontology has acquired over time several meanings, and it has also been approached in many different ways, but all of these are connected to the concepts of an *'objective existence'* and categories of items. A related and also important function of Ontology is to *classify and/or categorize* items and essential aspects of reality [2],[206]-[210]. Mathematicians specialised in Group Theory are very familiar with the classification problem into various types of the mathematical objects, or structures called 'groups'. Computer scientists that carry out ontological classifications, or study AI and Cognitive Science [201], are also interested in the logical foundations of computer science [1],[194],[197],[201]. We shall thus employ the adjective *"ontological"* with the meaning of pertaining to objective, real existence in its essential aspects. We shall also consider here the noun *existence* as a basic, or primary concept which cannot be defined in either simpler or atomic terms, with the latter in the sense of Wittgenstein. The authors aim at a concise presentation of novel methodologies for studying such difficult, as well as controversial, ontological problems of Space and Time at different levels of objective reality defined here as Complex, Super–Complex and Ultra–Complex Dynamic Systems, simply in order 'to divide and conquer'. The latter two are biological organisms, human (and perhaps also hominide) societies, and more generally, variable 'systems' and meta-systems that are not recursively–computable. An attempt is made from the viewpoint of the recent theory of ontological levels [2],[40],[137],[206]-[209] to understand the origins and emergence of life, the dynamics of the evolution of organisms and species, the ascent of man and the co-emergence, as well as co-evolution of human consciousness within organised societies. It is also attempted here to classify more precisely the levels of reality and species of organisms than it has been thus far reported.

In spite of the difficulties associated with understanding the essence of life, the human mind, consciousness and its origins, one can define pragmatically the human brain in terms of its neurophysiological functions, anatomical and microscopic structure, but one cannot as readily observe and define the much more elusive human mind. The existence of the human mind depends both upon a fully functional human brain and its training or education by the human society. Human minds that do not but weakly interact with those of any other member of society are partially disfunctional, thus creating difficult problems with the society integration of such humans. Obviously, it does take a fully functional mind to observe and understand the human mind. Theories of the mind are thus considered here in the context of a novel ontological theory of levels. Thus, in this monograph we have focused in the last two sections on the ultra-complex problems raised by human consciousness and its co-emergence with the human society, as well as on the very complex interactions in modern society and their possible outcomes. Current thinking [87], [91],[182],[186],[188], [190],[195]-[196],[203],[247] considers the actual emergence of human consciousness [83],[91],[186],[190],[261] –and also its ontic category– to be critically dependent upon the existence of both a human society level of *minimal* (tribal) organization [91],[186],[190], and that of an extremely complex structuraln –functional unit –the human brain with an *asymmetric* network topology and a dynamic network connectivity of very high-order [187],[218], [262]. Anticipatory systems and complex causality at the top levels of reality are also discussed in the context of Complex Systems Biology (CSB), psychology, sociology and ecology.

Our novel approach to meta-systems and levels using Category Theory and HDA mathematical representations is also applicable–albeit in a modified form–to supercomputers, complex quantum computers, man–made neural networks and novel designs of advanced artificial intelligence (AI) systems (AAIS).

The next six sections proceed from the Ontological Theory of Levels to Categorical Ontology, the definition and classification of dynamic systems, the emergence of complex systems, the origins of Life on Earth, the emergence of super-complex organisms through evolution , as well as the co-emergence and co-evolution of *H. sapiens* and society. Section 8 is a concise presentation of novel designs of meta-level AI systems, as well as Biomimetics, in general. Rigorous definitions of the logical and mathematical concepts employed in this monograph are provided in Sections 2 to 7, and also in the Appendix. A step-by-step construction of our conceptual framework was provided in a recent series of publications on categorical ontology of levels and complex systems dynamics [33]-[34],[39]-[40], and are here also summarized in Sections 2 to 4. Besides introducing super-complex and ultra-complex systems that emerge as meta-levels of ontic reality, the ontological classification of dynamic systems is considered in Section 4 from the point of view of dynamic analogy, topological conjugacy and dynamic adjointness beween systems. Classes of dynamically equivalent systems lead directly to a certain type of groupoids associated with systems dynamics and their symmetry, and are therefore called *dynamic groupoids*; categories of dynamic groupoids and their homomorphisms are called therefore *dynamic categories*. Section 5 begins with a brief, theoretical subsection on Complex Systems Biology (CSB), and a subsection on general biological principles and postulates; the next three subsections discuss the emergence of life and consider the problem of mathematical representations of functional organisms, including the original life-form on Earth, called the *primordial*. The next five subsections in Section 5 present several detailed examples of fully developed mathematical representations of functional organisms in categories, such as: (Metabolic-Repair)-systems, (M, R)-systems, Łukasiewics logic-algebra representation of dynamic genetic networks, and dynamic programming/algebraic geometry models of oncogenesis and cellular controls. The last subsections in Section 5 are introducing two general representations of dynamic processes in evolutionary biology- autopoiesis and chains/compositions of local procedures (COLPs) that represent speciation and the emergence of complex species as possible solutions to local-to-global, dynamic problems of evolutionary biology. Section 6 presents a literature consensus regarding the co-emergence and co-evolution of human consciousness and society, even though the empirical/historical evidence is scarce, incomplete and often debated. For us the most interesting question by far is how human consciousness and civilisation emerged subsequent only to the emergence of *H. sapiens*. This may have arisen through the development of speech-syntactic language and an appropriately organized 'primitive' society [91],[186] (perhaps initially made of hominins/hominides). No doubt, the details of this highly complex, emergence process have been the subject of intense controversies over the last several centuries, and many differing opinions, even among these authors, and they will continue to elude us since much of the essential data must remains either scarce or unattainable. Defining human consciousness proves to be an even more difficult task than defining super-complex systems which represent functional organisms. It is also suggested in Section 6 that without the consideration of meta-level processes of neurophysiological subprocesses in the human brain, as well as their representations in higher dimensional algebra (HDA), one may not be able to obtain an improved understanding of human consciousness. Moreover, the subtle and most complex interactions present in human societies required the introduction in Section 7 of several new concepts in order to represent certain essential aspects of society dynamics and evolution such as those related to memes and political decision making. The continuation of the very existence of human society may now depend upon an improved understanding of highly complex systems and the human mind, and also upon how the global human society interacts with the rest of the biosphere and its natural environment. It is most likely that such tools that we shall suggest here might have value not only to the sciences of complexity and Ontology but, more generally also, to all philosophers seriously interested in keeping on the rigorous side of the fence in their arguments. Following Kant's critique of 'pure' reason and Wittgenstein' s critique of language misuse in philosophy, one needs also to critically examine the possibility of using general and universal, mathematical language and tools in formal approaches to a rigorous, formal Ontology. Throughout this monograph we shall use the attribute *'categorial'* only for philosophical and linguistic arguments. On the other hand, we shall utilize the rigorous term *'categorical'* only in conjunction with applications of concepts and results from the more restrictive, but still quite general, mathematical *Theory of Categories, Functors and Natural Transformations* (TC-FNT) presented here concisely in Section 3 . According to SEP (2006): "Category theory ... is a general mathematical **theory of structures and of systems of structures**. *Category theory is both an interesting object of philosophical study, and a potentially powerful formal tool for philosophical investigations of concepts such as space, system, and even truth... It has come to occupy a central position in contemporary mathematics and theoretical computer science, and is also applied to mathematical physics.*" [248]. Traditional, modern philosophy– considered as a search for improving knowledge and wisdom– does also aims at unity that might be obtained as suggested by Herbert Spencer in 1862 through a *'synthesis of syntheses'*; this could be perhaps iterated many times because each treatment is based upon a critical evaluation and provisional improvements of previous treatments or stages. One notes however that this methodological question is hotly debated by modern philosophers beginning, for example, by Descartes before Kant and Spencer; Descartes championed with a great deal of success the *'analytical'* approach in which *all* available evidence is, in principle, examined critically and skeptically first both by the proposer of novel metaphysical claims and his, or her, readers. Descartes equated the 'synthetic' approach with the Euclidean 'geometric' (axiomatic) approach, and thus relegated synthesis to a secondary, perhaps less significant, role than that of critical *analysis* of scientific 'data' input, such as the laws, principles, axioms and theories of all specific sciences. Spinoza's, Kant's and Spencer's styles might be considered to be synthetic by Descartes and all Cartesians, whereas Russell's approach might also be considered to be analytical. Clearly and correctly, however, Descartes did not regard analysis (A) and synthesis (S) as exactly inverse to each other, such as $A \rightleftarrows S$, and also not merely as 'bottom–up' and 'top–bottom' processes ($\downarrow\uparrow$). Interestingly, unlike Descartes' discourse of the philosophical method, his treatise of philosophical principles comes closer to the synthetic approach in having definitions and deductive attempts, logical inferences, not unlike his 'synthetic' predecessors, albeit with completely different claims and perhaps a wider horizon. The reader may immediately note that if one, as proposed by Descartes, begins the presentation or method with an analysis A, followed by a synthesis S, and then reversed the presentation in a follow-up treatment by beginning with a synthesis $S*$ followed by an analysis A' of the predictions made by S' consistent, or analogous, with A, then obviously $AS \neq S'A'$ because we assumed that $A \simeq A'$ and that $S \neq S'$. Furthermore, if one did not make any additional assumptions about analysis and synthesis, then *analysis* $\rightarrow$ *synthesis* $\neq$ *synthesis* $\rightarrow$ *analysis*, or $AS \neq SA$, that is analysis and synthesis obviously *'do not commute'*; such a theory when expressed mathematically would be then called *'non-Abelian'*. This is also a good example of the meaning of the term non-Abelian in a philosophical, epistemological context.

2. The Theory of Levels in Ontology

This section outlines our novel methodology and approach to the ontological theory of levels, which is then applied in subsequent sections in a manner consistent with our recently published developments [33]-[34],[39]-[40]. Here, we are in harmony with the theme and approach of Poli's ontological theory of levels of reality [121], [206]–[211]) by considering both philosophical–categorial aspects such as Kant's relational and modal categories, as well as categorical–mathematical tools and models of complex systems in terms of a dynamic, evolutionary viewpoint.

2.1. Fundamentals of Poli's Theory of Levels

The ontological theory of levels by Poli [206]-[211] considers a hierarchy of *items* structured on different levels of reality, or existence, with the higher levels *emerging* from the lower, but usually *not* reducible to the latter, as claimed by widespread reductionism. This approach modifies and expands considerably earlier work by Hartmann [137] both in its vision and the range of possibilities. Thus, Poli in [206]-[211] considers four realms or *levels* of reality: Material-inanimate/Physico-chemical, Material-living/Biological, Psychological and Social. Poli in [211] has stressed a need for understanding *causal and spatiotemporal* phenomena formulated within a *descriptive categorical context* for theoretical levels of reality. There is the need in this context to develop a *synthetic* methodology in order to compensate for the critical ontic data analysis, although one notes (cf. Rosen in 1987 [232]) that analysis and synthesis are not the exact inverse of each other. At the same time, we address in categorical form the *internal dynamics*, the *temporal rhythm, or cycles*, and the subsequent unfolding of reality. The genera of corresponding concepts such as 'processes', 'groups', 'essence', 'stereotypes', and so on, can be simply referred to as *'items'* which allow for the existence of many forms of causal connection [210]-[211]. The implicit meaning is that the *irreducible multiplicity* of such connections converges, or it is ontologically integrated within a *unified synthesis.*

2.2. Towards a Formal Theory of Levels in Ontology

This subsection will introduce in a concise manner fundamental concepts of the ontological theory of levels. Further details were reported by Poli in [206]-[211], and by Baianu and Poli in this volume [40].

2.3. The Object-based Approach vs Process-based (Dynamic) Ontology

In classifications, such as those developed over time in Biology for organisms, or in Chemistry for chemical elements, the *objects* are the basic items being classified even if the 'ultimate' goal may be, for example, either evolutionary or mechanistic studies. An ontology based strictly on object classification may have little to offer from the point of view of its cognitive content. It is interesting that D'Arcy W. Thompson arrived in 1941 at an ontologic "*principle of discontinuity*" which "is inherent in all our classifications, whether mathematical, physical or biological... In short, nature proceeds *from one type to another* among organic as well as inorganic forms... and to seek for stepping stones across the gaps between is to seek in vain, for ever." (p.1094 of Thompson in [259], re-printed edition). Whereas the existence of different ontological levels of reality is well-established, one cannot also discard the study of emergence and co-emergence processes as a path to improving our understanding of the relationships among the ontological levels, and also as an important means of ontological classification. Furthermore, the emergence of ontological meta-levels cannot be conceived in the absence of the simpler levels, much the same way as the chemical properties of elements and molecules cannot be properly understood without those of their constituent electrons.

It is often thought that the *object-oriented* approach can be readily converted into a process-based one. It would seem, however, that the answer to this question depends critically on the ontological level selected. For example, at the quantum level, *object and process become inter-mingled*. Either comparing or moving between levels– for example through emergent processes– requires ultimately a *process-based* approach, especially in Categorical Ontology where relations and inter-process connections are essential to developing any valid theory. Ontologically, the quantum level is a fundamentally important starting point which needs to be taken into account by any theory of levels that aims at completeness. Such completeness may not be attainable, however, simply because an 'extension' of Gödel's theorem may hold here also. The fundamental quantum level is generally accepted to be dynamically, or intrinsically *non-commutative*, in the sense of the *non-commutative quantum logic* and also in the sense of *non-commuting quantum operators* for the essential quantum observables such as position and momentum. Therefore, any comprehensive theory of levels, in the sense of incorporating the quantum level, is thus –*mutatis mutandis*– *non-Abelian.* A paradigm shift towards a *non-Abelian Categorical Ontology* has already begun [33]-[34],[37]-[38],[40],[69], as it will be further explained in the next section.

2.4. From Component Objects and Molecular/Anatomical Structure to Organismic Functions and Relations: A Process–Based Approach to Ontology

Wiener in 1950 made the important remark that implementation of *complex functionality* in a (complicated, but not necessarily complex–in the sense defined above) machine requires also the design and construction of a correspondingly *complex structure*, or structures [269]. A similar argument holds *mutatis mutandis*, or by induction, for *variable* machines, variable automata and variable dynamic systems [12]-[23]; therefore, if one represents organisms as variable dynamic systems, one *a fortiori* requires a *super-complex structure* to enable or entail *super-complex dynamics*, and indeed this is the case for organisms with their extremely intricate structures at both the molecular and *supra-molecular* levels. This seems to be a key point which appears to have been missed in the early-stages of Robert Rosen's theory of simple (M, R)-systems, prior to 1970, that were deliberately designed to have "no structure" as it was thought they would thus attain the highest degree of generality or abstraction, but were then shown by Warner to be equivalent to a special type of sequential machine or classical automaton [17],[264].

The essential properties that define the super– and ultra– complex systems derive from the *interactions, relations and dynamic transformations* that are ubiquitous at such levels of reality– which need to be distinguished from the levels of organization internal to any biological organism or biosystem. Therefore, a complete approach to Ontology should obviously include *relations and interconnections* between items, with the emphasis on *dynamic processes, complexity* and *functionality* of systems. This leads one to consider general relations, such as *morphisms* on different levels, and thus to the *categorical viewpoint* of Ontology. The *process-based approach* to an Universal Ontology is therefore essential to an understanding of the Ontology of Reality Levels, hierarchies, complexity, anticipatory systems, Life, Consciousness and the Universe(s). On the other hand, the opposite approach, based on objects, is perhaps useful only at the initial cognitive stages in experimental science, such as the simpler classification systems used for efficiently organizing data and providing a simple data structure. We note here also the distinct meaning of 'object' in psychology, which is much different from the one considered in this subsection; for example, an external process can be 'reflected' in one's mind as an 'object of study'. This duality, or complementarity between 'object' and 'subject', 'objective' and 'subjective' seems to be widely adopted in philosophy, beginning with Descartes and continuing with Kant, Heidegger, and so on. A somewhat similar, but not precisely analogous distinction is fundamental in standard Quantum Theory– the distinction between the observed/measured system (which is the quantum, 'subject' of the measurement), and the measuring instrument (which is a classical 'object' that carries out the measurement).

2.5. Physicochemical Structure–Function Relationships

It is generally accepted at present that structure-functionality relationships are key to the understanding of super-complex systems such as living cells and organisms. Integrating structure–function relationships into a Categorical Ontology approach is undoubtedly a viable alternative to any level reduction, and philosophical/epistemologic reductionism in general. Such an approach is also essential to the science of complex/super-complex systems; it is also considerably more difficult than either physicalist reductionism, entirely *abstract re-*

lationalism or 'rhetorical mathematics'. Moreover, because there are many alternative ways in which the physico-chemical structures can be combined within an organizational map or relational complex system, there is a *multiplicity of 'solutions'* or mathematical models that needs be investigated, and the latter are not computable with a digital computer in the case of complex/super-complex systems such as organisms [23],[232]. The problem is further compounded by the presence of *structural disorder* (in the physical structure sense) which leads to a very high *multiplicity* of dynamical-physicochemical structures (or 'configurations') of a biopolymer– such as a protein, enzyme, or nucleic acid, of a biomembrane, as well as of a living cell, that correspond to a single function or a small number of physiological functions [20]; this complicates the assignment of a 'fuzzy' physico-chemical structure to a well-defined biological function unless extensive experimental data are available, as for example, those derived through computation from 2D-NMR spectroscopy data (as for example by Wütrich, in 1996 [271]), or neutron/X-ray scattering and related multi-nuclear NMR spectroscopy/relaxation data [20] Detailed considerations of the ubiquitous, or universal, partial disorder effects on the structure-functionality relationships were reported for the first time by Baianu in 1980 [20]. Specific aspects were also recently discussed by Wütrich in 1996 on the basis of 2D-NMR analysis of 'small' protein configurations in solution [271].

As befitting the situation, there are devised *universal* categories of reality in its entirety, and also subcategories which apply to the respective sub-domains of reality. We harmonize this theme by considering categorical models of complex systems in terms of an evolutionary dynamic viewpoint using the mathematical methods of Category Theory which afford describing the characteristics, classification and emergence of levels, besides the links with other theories that are, *a priori*, essential requirements of any ontological theory. We also underscore a significant component of this essay that relates the ontology to geometry/topology; specifically, if a level is defined via 'iterates of local procedures' (cf 'items in iteration' cf. Brown and İçen in [71]), that will further expanded upon in the last sections; then we will have a handle on describing its intrinsic governing dynamics (with feedback). As we shall see in the next subsection, categorical techniques– which form an integral part of our ontological considerations– provide a means of describing a hierarchy of levels in both a linear and interwoven, or *entangled*, fashion, thus leading to the necessary bill of fare: emergence, higher complexity and open, non-equilibrium/irreversible systems. We must emphasize that the categorical methodology selected here is *intrinsically 'higher dimensional'*, and can thus account for meta–levels, such as 'processes between processes...' within, or between, the levels–and sub-levels– in question. Whereas a strictly Boolean classification of levels allows only for the occurrence of *discrete* ontological levels, and also does not readily accommodate either *contingent* or *stochastic sub-levels*, the LM-logic algebra is readily extended to *continuous*, *contingent* or even *fuzzy* sub-levels, or levels of reality [11],[23],[32]-[34],[39]-[40],[120],[140]. Clearly, a Non-Abelian Ontology of Levels would require the inclusion of either Q- or LM- logics algebraic categories (discussed in the following section) because it begins at the fundamental quantum level –where Q-logic reigns– and 'rises' to the emergent ultra-complex level(s) with 'all' of its possible sub-levels represented by certain LM-logics. (Further considerations on the meta–level question are presented by Baianu and Poli in this volume [40]). On each level of the ontological hierarchy there is a significant amount of connectivity through inter-dependence, interactions or general relations often giving rise to complex patterns that are not readily analyzed by partitioning or through stochastic methods as they are neither simple, nor are they random connections. This ontological situation gives rise to a wide variety of networks, graphs, and/or mathematical categories, all with different connectivity rules, different types of activities, and also a hierarchy of super-networks of networks of subnetworks. Then, the important question arises *what types of basic symmetry or patterns* such super-networks of items can have, and how do the effects of their sub-networks percolate through the various levels. From the categorical viewpoint, these are of two basic types: they are either *commutative* or *non-commutative*, where, at least at the quantum level, the latter takes precedence over the former, as we shall further discuss and explain in the following sections.

We are presenting next a Categorical Ontology of highly complex systems, discussing the modalities and possible operational logics of living organisms, in general.

3. Categorical Ontology and Categorical Logics

3.1. *Basic Structure of Categorical Ontology. The Theory of Levels: Emergence of Higher Levels, Meta–Levels and Their Sublevels*

With the provisos specified above, our proposed methodology and approach employs concepts and mathematical techniques from Category Theory which afford describing the characteristics and binding of ontological levels besides their links with other theories. Whereas Hartmann in 1952 stratified levels in terms of the four frameworks: physical, 'organic'/biological, mental and spiritual [137], we restrict here mainly to the first three. The categorical techniques which we introduce provide a powerful means for describing levels in both a linear and interwoven fashion, thus leading to the necessary bill of fare: emergence, complexity and open non-equilibrium, or irreversible systems. Furthermore, any effective approach to Philosophical Ontology is concerned with *universal items* assembled in categories of objects and relations, involving, in general, transformations and/or processes. Thus, Categorical Ontology is fundamentally dependent upon both space and time considerations. Therefore, one needs to consider first a dynamic classification of systems into different levels of reality, beginning with the physical levels (including the fundamental quantum level) and continuing in an increasing order of complexity to the chemical–molecular levels, and then higher, towards the biological, psychological, societal and environmental levels. Indeed, it is the principal tenet in the theory of levels that : *"there is a two-way interaction between social and mental systems that impinges upon the material realm for which the latter is the bearer of both"* [209]. Therefore, any effective Categorical Ontology approach requires, or generates–in the constructive sense–a ***'structure'*** or *pattern of linked items* rather than a discrete set of items. The evolution in our universe is thus seen to proceed from the level of 'elementary' quantum 'wave–particles', their interactions *via* quantized fields (photons, bosons, gluons, etc.), also including the quantum gravitation level, towards aggregates or categories of increasing complexity. In this sense, the classical macroscopic systems are defined as 'simple' dynamical systems that are *computable recursively* as numerical solutions of mathematical systems of either ordinary or partial differential equations. Underlying such mathematical systems is always the Boolean, or chrysippian, logic, namely, the logic of sets, Venn diagrams, digital computers and perhaps automatic reflex movements/motor actions of animals. The simple dynamical systems are always recursively computable (see for example, Suppes, 1995–2006 [253]-[254], and also [23]), and in a certain specific sense, both degenerate and *non-generic*, and consequently also they are *structurally unstable* to small perturbations; such systems are, in general, deterministic in the classical sense, although there are arguments about the possibility of chaos in quantum systems. The next higher order of systems is then exemplified by 'systems with chaotic dynamics' that are conventionally called 'complex' by physicists who study 'chaotic' dynamics/Chaos theories, computer scientists and modelers even though such physical, dynamical systems are still completely deterministic. It has been formally proven that such 'systems with chaos' are *recursively non-computable* (see for example, refs. [23] and [28] for a 2-page, rigorous mathematical proof and relevant references), and therefore they cannot be completely and correctly simulated by digital computers, even though some are often expressed mathematically in terms of iterated maps or algorithmic-style formulas. Higher level systems above the chaotic ones, that we shall call *'super–complex, biological systems'*, are the living organisms, followed at still higher levels by the *ultra-complex 'systems'* of the human mind and human societies that will be discussed in the last sections. The evolution to the highest order of complexity- the ultra-complex, meta–'system' of processes of the human mind–may have become possible, and indeed accelerated, only through human societal interactions and effective, elaborate/rational and symbolic communication through speech (rather than screech –as in the case of chimpanzees, gorillas, baboons, etc).

Then, we consider briefly those integrated functions of the human brain that support the ultra-complex human mind and its important roles in societies. Mores specifically, we propose to combine a critical analysis of language with precisely defined, abstract categorical concepts from Algebraic Topology reported by Brown et al, in 2007 [69], and the general-mathematical Theory of Categories, Functors and Natural Transformations: [56], [80], [98]-[102], [105]-[106],[113],[115-[119],[130], [133]-[135],[141]-[143], [151],[154], [161]-[163],[165]-[168], [172], [175]-[177],[183], [192]-[194],[198]-[199] [213]-[215],[225], [227],[246], [252], [256] into a categorical framework which is suitable for further ontological development, especially in the relational rather than modal ontology of complex spacetime structures. Basic concepts of Categorical Ontology are presented in this section, whereas formal definitions are relegated to one of our recent, detailed reports [69]. On the one hand, philosophical categories according to Kant are: *quantity, quality, relation* and *modality*, and the most complex and far-reaching questions concern the relational and modality-related categories. On the other hand, mathematical categories are considered at present as the most general and universal structures in mathematics, consisting of related *abstract objects connected by arrows.* The abstract objects in a category may, or may not, have a specified *structure*, but must all be of the same type or kind in any given category. The arrows (also called *'morphisms'*) can represent relations, mappings/functions, operators, transformations, homeomorphisms, and so on, thus allowing great flexibility in applications, including those outside mathematics as in: Logics [118]-[120], Computer Science [1], [161]-[163] [201],[248], [252], Life Sciences [5],[11]-[17],[19],[23],[28]-[36],[39],[40],[42]-[44],[70],[74],[103]-[104],[230],[232],[234]-[238],[264], Psychology, Sociology [33],[34],[39],[40],[74], and Environmental Sciences [169]. The mathematical category also has a form of *'internal' symmetry*, specified precisely as the *commutativity* of chains of morphism compositions that are unidirectional only, or as *naturality of diagrams* of morphisms; finally, any object A of an abstract category has an associated, unique, identity, 1_A, and therefore, one can replace all objects in abstract categories by the identity morphisms. When all arrows are *invertible*, the special category thus obtained is called a *'groupoid'*, and plays a fundamental role in the field of mathematics called Algebraic Topology.

The categorical viewpoint– as emphasized by William Lawvere, Charles Ehresmann and most mathematicians– is that the key concept and mathematical structure is that of *morphisms* that can be seen, for example, as abstract relations, mappings, functions, connections, interactions, transformations, and so on. Thus, one notes here how the philosophical category of *'relation'* is closely allied to the basic concept of morphism, or arrow, in an abstract category; the implicit tenet is that *arrows are what counts.* One can therefore express all essential properties, attributes, and structures by means of arrows that, in the most general case, can represent either philosophical 'relations' or modalities, the question then remaining if philosophical–categorial properties need be subjected to the categorical restriction of *commutativity.* As there is no *a priori* reason in either nature or 'pure' reasoning, including any form of Kantian 'transcendental logic', that either relational or modal categories should in general have any symmetry properties, one cannot impose onto philosophy, and especially in ontology, all the strictures of category theory, and especially commutativity. Interestingly, the same comment applies to Logics: only the simplest forms of Logics, the Boolean and intuitionistic, Heyting-Brouwer logic algebras are commutative, whereas the algebras of many-valued (MV) logics, such as Łukasiewicz logic are *non-commutative* (or *non-Abelian*).

3.2. *Categorical Representations of the Ontological Theory of Levels: From Simple to Super– and Ultra– Complex Dynamic Systems. Abelian vs. Non-Abelian Theories*

General system analysis seems to require formulating ontology by means of categorical concepts (Baianu and Poli, 2010 [40]; Brown et al.[69]). Furthermore, Category Theory appears as a natural framework for any general theory of transformations or dynamic processes, just as Group Theory provides the appropriate framework for classical dynamics and quantum systems with a finite number of degrees of freedom. Therefore, we have adopted a categorical approach as the starting point, meaning that we are looking for *"what is universal"* (in some domain, or in general), and that only for simple systems this involves *commutative* modelling diagrams and structures (as, for example, in Figure 1 of Rosen, 1987 [232]). Note that this ontological use of the word *'universal'* is quite distinct from the mathematical use of *'universal property'*, which means that a property of a construction on particular objects is defined by its relation to *all* other objects (i.e., it is a *global* attribute), usually through constructing a morphism, since this is the only way, in an *abstract* category, for objects to be related. With the first (ontological) meaning, the most universal feature of reality is that it is *temporal*, i.e. it changes, it is subject to countless transformations, movements and alterations. In this select case of *universal temporality*, it seems that the two different meanings can be brought into superposition through appropriate formalization. Furthermore, *concrete* categories may also allow for the representation of ontological 'universal items' as in certain previous applications to categories of neural networks [14],[23],[32]-[33]. For general categories, however, each object is a kind of a Skinnerian black box, whose only exposure is through input and output, i.e. the object is given by its *connectivity* through various morphisms, to other objects. For example, the dual of the category of sets still has objects but these have *no structure* (from the categorical viewpoint). Other types of category are important as expressing useful relationships on structures, for example *lextensive* categories, which have been used to express a general van Kampen theorem by Brown and Janelidze in 1997 [65].

Thus, abstract mathematical structures are developed to define *relationships*, to deduce and calculate, to exploit and define analogies, since *analogies are between relations* between things rather than between things themselves. A description of a new structure is in some sense a development of part of a *new language*; the notion of structure is also related to the notion of *analogy*. It is one of the triumphs of the mathematical theory of categories in the 20th century to make progress towards *unifying* mathematics through the finding of *analogies* between various behavior of structures across different areas of mathematics. This theme is further elaborated in the article by Brown and Porter in 2006 [66] who argued that many analogies in mathematics, and in many other areas, are *not* between objects themselves but *between the relations* between objects.

3.3. *Categorical Logics of Processes and Structures: Universal Concepts and Properties*

The logic of classical events associated with either mechanical systems, mechanisms, universal Turing machines, automata, robots and digital computers is generally understood to be simple, *Boolean* logic. The same applies to Einstein's GR. It is only with the advent of quantum theories that quantum logics of events were introduced which are *non-commutative*, and therefore, also *non-Boolean*. Somewhat surprisingly, however, the connection between quantum logics (QL) and other *non-commutative* many-valued logics, such as the Łukasiewicz logic, has only been recently made [88],[31]–[34].

Such considerations are also of potential interest for a wide range of complex systems, as well as quantum ones, as it has been pointed out previously [18],[23],[31]-[34]. Furthermore, both the concept of 'Topos' and that of variable category, can be further generalized by the involvement of *many-valued* logics, as for example in the case of 'Łukasiewicz-Moisil, or LM Topos' [32]. This is especially relevant for the development of *non-Abelian dynamics* of complex and super-complex systems; it may also be essential for understanding human consciousness in the sequel.

3.4. *Quantum Logics (QL), Logical Lattice Algebras (LLA) and Łukasiewicz Quantum Logic (LQL)*

As pointed out by Birkhoff and von Neumann in 1936, a logical foundation of quantum mechanics consistent with quantum algebra is essential for the internal consistency of the theory. Such a non-traditional logic was initially formulated by Birkhoff and von Neumann in 1936 [52], and then called 'Quantum Logic' (or subsequently Q-logics). Subsequent research on Quantum Logics [88] resulted in several approaches that involve several types of non-distributive lattice (algebra) for n–valued quantum logics. Thus, modifications of the Łukasiewicz Logic Algebras that were introduced in the context of algebraic categories by Georgescu and Popescu in 1968 [119], followed by Georgescu and Vraciu in 1970 with a characterization of LM-algebras [118], also recently being reviewed and expanded by Georgescu [120] , can provide an appropriate framework for representing quantum systems, or– in their unmodified form- for describing the activities of complex networks in categories of Łukasiewicz Logic Algebras [18]. There is a logical inconsistency however between the quantum algebra and the Heyting logic algebra of a standard topos as a candidate for quantum logic [32]–[34],[88].

Furthermore, quantum algebra and topological approaches that are ultimately based on set-theoretical concepts and differentiable spaces (manifolds) also encounter serious problems of internal inconsistency. There is a basic logical inconsistency between quantum logic–which is not Boolean–and the Boolean logic underlying all differentiable manifold approaches that rely on continuous spaces of points, or certain specialized sets of elements. A possible solution to such inconsistencies is the definition of a generalized 'topos'–like concept, such as a *Quantum, Extended Topos* concept which is consistent with both Quantum Logic and Quantum Algebras [3],[164], being thus suitable as a framework for unifying quantum field theories and modelling in complex systems biology.

3.5. *Lattices and von Neumann-Birkhoff (VNB) Quantum Logic [52]: Definition and Some Logical Properties.*

We commence here by giving the set-based definition of a lattice.

D1. An *s–lattice* **L**, or a *'set-based' lattice*, is defined as a partially ordered set that has all binary products (defined by the *s*–lattice operation " $\bigwedge$") and coproducts (defined by the *s*–lattice operation "$\bigvee$ "), with the "partial ordering" between two elements X and Y belonging to the *s*–lattice being written as "$X \preceq Y$". The partial order defined by $\preceq$ holds in **L** as $X \preceq Y$ if and only if $X = X \bigwedge Y$ (or equivalently, $Y = X \bigvee Y$ Eq.(3.1) (p.49 of Mac Lane and Moerdijk's book [177]). A *lattice* can also be defined as a *category* subject to all ETAC axioms (see, for examplel [166])– but not subject, in general, to the Axiom of Choice usually encountered with sets relying on (distributive) Boolean Logic [12]-[18], [25]– as well as 'partial ordering' properties, $\preceq$.

3.6. *Łukasiewicz-Moisil (LM) Quantum Logic (LQL) and Algebras*

Quantum algebras, following Majid in 1995 and 2002 [178]-[179], involve detailed studies of the properties and representations of Quantum State Spaces (QSS; see for example, Alfsen and Schultz in 2003 [3]). As an example, with all truth 'nuances' or assertions of the type $\langle\langle$*system A* is excitable to the *i*-th level and system B is excitable to the *j*-th level$\rangle\rangle$ one can define a special type of lattice that subject to the axioms introduced by Georgescu and Vraciu [118] becomes a *n-valued Łukasiewicz-Moisil, or LM–, Algebra* ; for further details see also the subsection on LM-algebra in the **Appendix** . Further algebraic and logic details are provided by Georgescu in [120] and Baianu et al. in [32]. In order to have the *n*-valued Łukasiewicz Logic Algebra represent correctly the observed behaviours of quantum systems (that involve a quantum system interactions with a measuring instrument –which is a macroscopic object) several of the LM–algebra axioms have to be significantly changed so that the resulting lattice becomes *non-distributive* and also (possibly) *non–associative* [88]. With an appropriately defined quantum logic of events one can proceed to define Hilbert and von Neumann/ C^*–algebras, etc, in order to be able to utilize the 'standard' procedures of quantum theories (precise definitions of these fundamental quantum algebraic concepts were presented in [6]. On the other hand, for classical systems, modelling with the unmodified Łukasiewicz Logic Algebra can also include both stochastic and fuzzy behaviours. For an example of such models the reader is referred to a previous publication modelling the activities of complex genetic networks from a classical standpoint [18]. One can also define as in [118] the 'centers' of certain types of LM, *n*-valued Logic Algebras; then one has the following important theorem for such Centered Łukasiewicz n-Logic Algebras which actually defines an equivalence relation.

Theorem 0.1. The Adjointness Theorem (Georgescu and Vraciu, 1970 in ref. [118]).
*There exists an Adjointness between the Category of Centered Łukasiewicz n-Logic Algebras, **CLuk**–n, and the Category of Boolean Logic Algebras (**Bl**).*

Remark 0.1. This adjointness (in fact, actual equivalence) relation between the Centered Łukasiewicz n-Logic Algebra Category and **Bl** has a logical basis: $non(non(A)) = A$ in both **Bl** and **CLuk**–n.

Remark 0.2. The natural equivalence logic classes defined by the adjointness relationships in the above Adjointness Theorem define a fundamental, *'logical groupoid'* structure.

Remark 0.3. In order to adapt the standard Łukasiewicz Logic Algebra to the appropriate Quantum Łukasiewicz Logic Algebra, LQL, a few axioms of LM-algebra need modifications, such as : $N(N(X)) = Y \neq X$ (instead of the restrictive identity $N(N(X)) = X$, whenever the context, or 'measurement preparation' interaction conditions for quantum systems are incompatible with the standard 'negation' operation N of the Łukasiewicz Logic Algebra; the latter remains however valid for the operation/ dynamics of classical or semi–classical systems, such as various complex networks with n-states (cf. Baianu in 1977 [18],[23]). Further algebraic and conceptual details are provided in a rigorous review by Georgescu in [120], and also in two recently published reports [33],[69].

3.7. *A Hierarchical, Formal Theory of Levels. Commutative and Non-Commutative Structures: Abelian Category Theory vs. Non-Abelian Theories*

Ontological classification based on items involves the organization of concepts, and indeed theories of knowledge, into a *hierarchy of categories of items at different levels of 'objective reality'*, as reconstructed by scientific minds through either a *bottom-up* (induction, synthesis, or abstraction) process, or through a *top-down* (deduction) process [209], which proceeds from abstract concepts to their realizations in specific contexts of the 'real' world. Both modalities can be developed in a categorical framework. We discuss here only the bottom-up modality in Categorical Ontology.

One of the major goals of category theory is to see how the properties of a particular mathematical structure, say S, are reflected in the properties of the category $\mathsf{Cat}(S)$ of all such structures and of morphisms between them. Thus, the first step in category theory is that a definition of a structure should come with a definition of a morphism of such structures. Usually, but not always, such a definition is obvious. The next step is to compare structures. This might be obtained by means of a *functor* $A : \mathsf{Cat}(S) \longrightarrow \mathsf{Cat}(T)$. Finally, we want to compare such functors $A, B : \mathsf{Cat}(S) \longrightarrow \mathsf{Cat}(T)$. This is done by means of a natural transformation $\eta : A \Rightarrow B$. Here η assigns to each object X of $\mathsf{Cat}(S)$ a morphism $\eta(X) : A(X) \longrightarrow B(X)$ satisfying a commutativity condition for any morphism $a : X \longrightarrow Y$. In fact we can say that η assigns to each morphism a of $\mathsf{Cat}(S)$ a commutative square of morphisms in $\mathsf{Cat}(T)$ (as shown in Diagram 13.2 of Brown, Glazebrook and Baianu in [69]). This notion of *natural transformation* is at the heart of category theory. As Eilenberg-Mac Lane wrote: "*to define natural transformations one needs a definition of functor, and to* *define the latter one needs a definition of category".* Also, the reader may have already noticed that 2-arrows become '3-objects' in the meta–category, or '3-category', of functors and natural transformations [69].

One could formalize-for example as outlined by Baianu and Poli in [40]–the hierarchy of multiple-level relations and structures that are present in biological, environmental and social systems in terms of the mathematical Theory of Categories, Functors and Natural Transformations (TC-FNT, see [69]). On the first level of such a hierarchy are the links between the system components represented as *'morphisms'* of a structured category which are subject to several axioms/restrictions of Category Theory, such as *commutativity* and associativity conditions for morphisms, functors and natural transformations. Then, on the second level of the hierarchy one considers *'functors'*, or links, between such first level categories, that compare categories without 'looking inside' their objects/system components. On the third level, one compares, or links, functors using *'natural transformations'* in a 3-category (meta-category) of functors and natural transformations. At this level, natural transformations not only compare functors but also look inside the first level objects (system components) thus 'closing' the structure and establishing 'the universal links' between items as an integration of both first and second level links between items. Note, however, that in general categories the objects have no 'inside', though they may do so for example in the case of 'concrete' categories.

From the point of view of mathematical modelling, the mathematical theory of categories models the dynamical nature of reality by representing temporal changes through either *variable* categories or through *toposes*. According to Mac Lane and Moerdijk in ref.[177] (p.1 of the Prologue), and also in refs.: [1],[21]-[22],[151],[165], and [252] certain variable categories can also be generated as a topos:

"A startling aspect of topos theory is that it unifies two seemingly wholly distinct mathematical subjects : on the one hand, topology and algebraic geometry, and on the other hand, logic and set theory. Indeed a topos *can be considered both as a "generalized space" and as a "generalized universe of sets". These different aspects arose independently around 1963 : with A. Grothendieck in his reformulation of sheaf theory for algebraic geometry, with William F. Lawvere in his search for an axiomatization of the category of sets and that of "variable" sets, and with Paul Cohen in the use of forcing to construct new models of Zermelo-Fraenkel set theory. The study of cohomology for generalized spaces led Grothendieck to define his notion of a topos. The cohomology was to be one with variable coefficients–for example, varying under the action of the fundamental group, as in N.E. Steenrod's work in algebraic topology, or more generally varying in a sheaf. "*

For example, the category of sets can be considered as a topos whose only generator is just a single point. A variable category of varying sets might thus have just a generator set. However, a qualitative distinction *does exist* between organisms–considered as complex systems– and 'simple', inanimate dynamical systems, in terms of the modelling process and the type of predictive mathematical models or representations that they can have [232], and also, previously in refs.[11]-[14],[22]-[24]. A relevant example of applications to the natural sciences, e.g., neurosciences, would be the higher-dimensional algebra representation of processes of cognitive processes of still more, linked sub-processes (Brown et al. [69], Brown and Porter [66]). Additional examples of the usefulness of such a categorical constructive approach to generating higher-level mathematical structures would be that of supergroups of groups of items, 2-groupoids, or double groupoids of items.

On the one hand, there is a second adjointness theorem concerning the category of fuzzy sets and a corresponding topos of sheaves:

Theorem 0.2. The Second Adjointness Theorem (published by Lawrence Neff Stout in 2004 [252]). *Let H be a completely distributive lattice, such as a Heyting logic algebra. Then there are pairs of adjoint functors between Goguen's category of fuzzy sets* ***Set(H)****, Eytan's logos* ***Fuz(H)*** *and the topos of sheaves on H,* ***Sh(H)****.*

On the other hand, the first **Ajointness Theorem** already discussed above establishes a natural equivalence between the category of centered Łukasiewicz logic algebras, **CLuk**–n, and **Bl**, the category of Boolean logic algebras. Because functional genomes of living organisms admit a Luk_n representation of genetic network activities but are not generally reducible to **CLuk**–n representations [18],[23], it follows that genomes do not admit a Boolean logic, complete representation as often attempted by digital 'genetic nets' or 'cell automata' models. *Mutatis mutandis* the same argument holds for the simple metabolic-replication, or (M, R)-systems that have equivalent automata representations [17],[264]. The interesting question then remains about the relationship between the category of Heyting algebras $\mathbf{H}_T$ and **Lukn**, the category of Luk_n- logic algebras. There is also the corresponding questions about the relationship between their representation categories: **Set(H)** for fuzzy systems, and $\mathbf{GNet}_{Lukn}$ for representations of functional genomes in living organisms; there are no known adjoint functor pairs between **Lukn** and $\mathbf{H}_T$, or **Set(H)**, of course. Therefore, even though relational models of physiologically functional organisms involve variable categories, or variable groupoids and variable topology (for example, variable gene or interactome network topology), as well as exhibit fuzzy behaviors [11]-[20], so far there is no strict topos of sheaves on a Heyting logic algebra, (and thus a completely distributive and commutative lattice) that has been found to possess an adequate representation of either functional organisms or genomes. On the other hand, we have previously reported that one can define an extended 'Topos', T_E, based on a Luk_n-logic algebra as an object classifier of T_E , which then admits representations of functional genetic network categories [32]. Naturally, such Luk_n-logic algebras are generally non-commutative, and their category, $\mathbf{GNet}_{Lukn}$ (as well as **Lukn** itself), is in general a *non-Abelian* category.

3.8. *Symmetry, Commutativity and Abelian Structures*

The hierarchy constructed above, up to level 3, can be further extended to higher, n-levels, always in a consistent, natural manner, that is using commutative diagrams. Let us see therefore a few simple examples or specific instances of commutative properties. The type of global, natural hierarchy of items inspired by the mathematical TC-FNT has a kind of *internal symmetry* because at all levels, the link compositions are *natural*, that is, if $f : x \longrightarrow y$ and $g : y \longrightarrow z \Longrightarrow h : x \longrightarrow z$, then the composition of morphism g with f is given by another unique morphism $h = g \circ f$. This general property involving the equality of such link composition chains or diagrams comprising any number of sequential links between the same beginning and ending objects is called *commutativity* (see for example Samuel and Zarisky, 1957 [241]), and is often expressed as a *naturality condition for diagrams.* This key mathematical property also includes the mirror-like symmetry $x \star y = y \star x$; when x and y are operators and the symbol '$\star$' represents the operator multiplication. Then, the equality of $x \star y$ with $y \star x$ defines the statement that "the x and y operators *commute*"; in physical terms, this translates into a sharing of the same set of eigenvalues by the two commuting operators, thus leading to 'equivalent' numerical results i.e., up to a multiplication constant); furthermore, the observations X and Y corresponding, respectively, to these two operators would yield the same result if X is performed before Y in time, or if Y is performed first followed by X. This property, when present, is very convenient for both mathematical and physical applications (such as those encountered in quantum mechanics). However, not all quantum operators 'commute', and not all categorical diagrams or mathematical structures are, or need be, commutative. *Non-commutativity* may therefore appear as a result of 'breaking' the 'internal symmetry' represented by commutativity. As a physical analogy, this might be considered a kind of *'symmetry breaking'* which is thought to be responsible for our expanding Universe and CPT violation, as well as many other physical phenomena such as phase transitions and superconductivity [267].

On the one hand, when commutativity is global in a structure, as in an Abelian (or commutative) group, commutative ring, etc., such a structure that is commutative throughout is usually called ***Abelian***.

However, in the case of category theory, this term 'Abelian' has been extended to a special class of categories that have meta-properties formally similar to those of the category **Ab-G** of commutative groups; the necessary and sufficient conditions for such 'Abelianness' of categories other than that of Abelian groups were expressed as three axioms **Ab1** to **Ab3** and their duals [113], as shown by Freyd in 1964; see also the details in [33] and [69]. Among such mathematical structures, *Abelian* categories have particularly interesting applications to rings and modules [117] and [213], in which case commutative diagrams are essential. Commutative diagrams are also being widely used in Algebraic Topology [61], [63], [68] , [183] and [246]. As one can see from both the earlier and more recent literature, Abelian categories have been studied in great detail, even though their study is far from complete.

On the other hand, the more general case is the *non-commutative* one. Several intriguing, 'non-commutative' or non-Abelian, examples are provided by certain *asymmetric* drawings by Escher, such as his perpetuum water mill, or his 3D-evading, illusory castle with monks 'climbing' from one level to the next–at 'same-height' (that might be considered as a hint to paradoxes caused by the imposition of only one level of reality, similar to Abbott's 'Flatland').

3.9. *Abelian Meta-Theorems*

Freyd in his 1964 CT book [113] has an interesting section on **meta**-theorems. In essence, all propositions or mathematical truth statements of a specific mathematical form "***p***" that are valid for the category of Abelian groups are also valid in any extended Abelian category defined by axioms Ab1 to Ab3 and their duals. Other types of meta-theorems are also possible for super-categories of categories, and of course such meta-theorems are not restricted to Abelian structures.

Thus, unlike most other mathematical theories, CT has statements about theorems that have stratified, higher levels of super- or meta - categories. Such meta-theories may prove useful in representing the higher levels of complexity as ontological *meta-levels*.

3.10. *Non-Abelian Theories and Spacetimes Ontology*

Any comprehensive Categorical Ontology theory is *a fortiori non-Abelian*, and thus recursively non-computable, on account of both the quantum level (which is generally accepted as being non-commutative), and the top ontological level of the human mind– which also operates in a non-commutative manner, albeit with a different, *multi-valued* logic than Quantum Logic. To sum it up, the operating/operational logics at both the top and the fundamental levels are *non-commutative* (the 'invisible' actor (s) who– behind the visible scene– make(s) both the action and play possible!). At the fundamental level, spacetime events occur according to a quantum logic (QL), or *Q-logic*, whereas at the top level of human consciousness, a different, non-commutative Higher Dimensional Logic Algebra prevails akin to the many-valued (Łukasiewicz - Moisil, or LM) logics of genetic networks [24], that were shown previously to exhibit non-linear, and also non-commutative/non-computable, biodynamics [18],[32]. Our viewpoint is that models constructed from category theory and higher dimensional algebra have potential applications towards creating a higher science of analogies which, in a descriptive sense, is capable of mapping imaginative subjectivity beyond conventional relations of complex systems. Of these, one may strongly consider a *generalized chronoidal–topos* notion that transcends the concepts of spatial–temporal geometry by incorporating *non-commutative multi–valued logic*. Current trends in the fundamentally new areas of quantum–gravity theories appear to endorse taking such a direction. We aim further to discuss some prerequisite algebraic–topological and categorical ontology tools for this endeavor, again relegating all rigorous mathematical definitions to the Brown, Glazebrook and Baianu [69]. It is interesting that Abelian Categorical Ontology (ACO) is also acquiring several new meanings and practical usefulness in the recent literature related to computer-aided (ontic/ontologic) classification, as in the case of: neural network categorical ontology in [14],[23],[103],[104], Genetic Ontology, Biological Ontology, and environmental representations by categories and functors of ultra-complex societies [169].

An example of a non-commutative structure relevant to Quantum Theory is that of the *Clifford algebra* of quantum observable operators (Dirac's textbook published in 1962 [93] ; see also Plymen and Robinson [205]), and also that of non-commutative quantum spaces in Quantum Field Theory [255]. Yet another- more recent and popular- example in the same QT context is that of C^*–algebras of (quantum) Hilbert spaces. Furthermore, the microscopic, or quantum, 'first' level of physical reality does *not* appear to be subject to the categorical naturality conditions of Abelian TC-FNT– the 'standard' mathematical theory of categories (functors and natural transformations). It would seem therefore that the commutative hierarchy discussed above is not sufficient for the purpose of a General, Categorical Ontology which considers all items, at all levels of reality, including those on the 'first', quantum level, which is non–commutative. On the other hand, the mathematical, Non-Abelian Algebraic Topology [68], the Non-Abelian Quantum Algebraic Topology (NA-QAT, in [38]), and the physical, Non-Abelian Gauge theories (NAGTs) may provide the ingredients for a proper foundation for non-Abelian, hierarchical multi-level theories of a super-complex system dynamics in a General Categorical Ontology (GCO). Furthermore, it was recently pointed out in refs.[36]-[39] that the current and future development of both NA-QAT and of a quantum-based Complex Systems Biology, *a fortiori*, involve *non-commutative*, many-valued logics of quantum events, such as a modified Łukasiewicz–Moisil (LMQ) logic algebra [32],[39], complete with a fully-developed, novel probability measure theory founded upon the LM-logic algebra [120]. The latter paves the way to a new projection operator theory founded upon the *non-commutative quantum logic of events*, or dynamic processes, thus opening the possibility of a complete, *non–Abelian quantum theory.* Furthermore, such recent developments point towards a paradigm shift in Categorical Ontology and to its extension to more general, *non-Abelian theories*, well beyond the bounds of commutative structures/spaces [255], and also free from the *logical* restrictions and limitations imposed by set theory [57],[59].

3.11. *Duality Concepts in Philosophy and Category Theory*
Duality and dual concepts are, and have been for a long time, the subject of philosophical investigations, including ontological ones. From the ancient concept of the 'complementary'– but– inseparable Yin–and–Yang to the more modern dualistic approaches to philosophy by Descartes or Hegel, dual concepts still hold a special attraction for the philosopher and mathematician who is concerned with the *unity* of nature and systems, be they natural or abstract/mathematical. Indeed, it would seem that *duality and adjointness* are at the heart of trends towards unity in mathematics [98]-[99],[166], and SEP-2006 [248] (including references cited therein). Like the two sides of a coin, both different/distinct/apposite and necessary, dual concepts are, according to Hegel, the very essence of dynamics and dialectics. In categories, CT duality is practically, and very simply, obtained by 'reversing the arrows' [183]. When all arrows are invertible in a category one has the natural structure of a *groupoid*, a structure that is fundamental in Topology [61], [63], [68]. Interestingly, most symmetric structures– as well as more generally– Abelian ones, are *self-dual*; likewise, the quantum operators representing observables are *self-adjoint*, and the Clifford algebra of Dirac's quantum theory [93]–which is noncommutative– is also *self-dual* [164]-[205]. Thus, the very extensive subject of duality deserves a detailed and thorough consideration which is however well beyond the scope of this monograph. Such duality considerations may very well lead to the fundamental structures of spacetime itself because space and time are fundamental, dual concepts that are joined together by the relativity of reference systems, and also tied up with the subtle nature of quantum gravity.

4. Systems, Dynamics and Complexity Levels

4.1. *Systems Classification in Ontology: Simple/Complex–Chaotic, Super–Complex and Ultra–Complex Systems viewed as Three Distinct Levels of Reality: Dynamic Analogy and Homology.*

We introduce here a few basic definition of a general, dynamical system that may facilitate further developments of the theory of levels in categorical ontology. No claim is here made however to either universality or mathematical rigour.

4.1.1. *Defining Dynamic Systems as Stable Spacetime Structures with Boundaries*

As defined by Baianu and Poli in this volume [40], a *system* is a dynamical (whole) entity able to maintain its working conditions; the system definition is here spelt out in detail by the following, general definition, **D1**.

D2. A *simple system* is in general a bounded, but not necessarily closed, entity– here represented as a category of stable, interacting components with inputs and outputs from the system's environment.

D3. More generally, at the *meta-level* of a *complex system* consisting of subsystems, or components, with internal boundaries among such subsystems, one needs to represent such a <*complex system of systems*> by a *super-category* (that is, as a *category of categories*, or 2-category).

As proposed by Baianu and Poli in [40], in order to define a *system* one therefore needs specify the following data: (1) components or subsystems, (2) mutual interactions or links; (3) a separation of the selected system by some boundary which distinguishes the system from its environment, without necessarily 'closing' the system to material exchange with its environment; (4) the specification of the system's environment; (5a) the specification of the system's categorical structure and dynamics; (5b) a super-category will be required only when either the components or subsystems need be themselves considered as represented by a category, i.e. the 'system' is, in fact, a *super-system of (sub)systems*, as it is indeed the case of all emergent *super-complex systems* or organisms. Also, as discussed by Baianu and Poli in [40], "the most general and fundamental property of a system is the *inter-dependence* of parts/components/sub-systems or variables."; *inter-dependence* is the presence of a certain organizational order in the relationship among the components or subsystems which make up the system. It can be shown that such organizational order must either result in a stable attractor or else it should occupy a stable spacetime domain, which is generally expressed in *closed* systems by the concept of equilibrium. On the other hand, in non-equilibrium, open systems, one cannot have a static but only a *dynamic self-maintenance* in a 'state-space region' of the open system – which cannot degenerate to either an equilibrium state or a single attractor spacetime region. Thus, non-equilibrium, open systems that are capable of self-maintenance (seen as a form of autopoiesis) will also be generic, or structurally-stable: their arbitrary, small perturbation from a homeostatic maintenance regime does not result either in completely chaotic dynamics with a single attractor or the loss of their stability. It may however involve an ordered process of changes - a process that follows a determinate pattern rather than random variation relative to the starting point. Systems are usually conceived as 'objects', or things, rather than processes even though at the core of their definition there are dynamic laws of evolution. Spencer championed in 1898 such evolutionary ideas/laws/principles not only in the biosphere but also in psychology and human societies. Furthermore, the usual meaning of 'dynamic systems' is associated with their treatments by a 'system' (array) of differential equations (either exact, ordinary or partial); note also that the latter case also includes 'complex' chaotic systems whose solutions cannot be obtained by recursive computation, for example with a digital computer or supercomputer.

4.1.2. *Selective Boundaries and Homeostasis. Varying Boundaries* vs *Horizons*

Boundaries are especially relevant to *closed* systems, although they also exist in many open systems. According to Poli [210]: "they serve to distinguish what is internal to the system from what is external to it", thus defining the fixed, overall structural topology of a closed system. By virtue of possessing boundaries, "a whole (entity) is something on the basis of which there is an interior and an exterior...which enables a difference to be established between the whole closed system and environment." (cf. Baianu and Poli, in this volume [40]). As proposed in [40], an essential feature of boundaries in open systems is that they can be crossed by matter. The boundaries of closed systems, however, cannot be crossed by molecules or larger particles. On the contrary, a horizon is something that one cannot reach. In other words, a horizon is not a boundary. This difference between horizon and boundary appears to be useful in distinguishing between systems and their environment.

One notes however that a boundary, or boundaries, may change or be quite selective, or directional–in the sense of dynamic fluxes crossing such boundaries–if the system is *open* and grows/develops as in the case of an organism, which will be thus characterized by a *variable* topology that may also depend on the environment, and is thus *context-dependent* as well. Perhaps the simplest example of a system that changes from *closed to open*, and thus has a *variable topology*, is that of a pipe equipped with a functional valve that allows flow in only one direction. On the other hand, a semi-permeable membrane such as a cellophane, thin-walled 'closed' tube– that allows water and small molecule fluxes to go through but blocks the transport of large molecules such as polymers through its pores– is *selective* and may be considered as a primitive/'simple' example of an open, selective system. Organisms, in general, are *open systems with variable topology* that incorporate both the valve and the selectively permeable membrane boundaries –albeit much more sophisticated and dynamic than the simple/fixed topology cellophane membrane–in order to maintain their stability and also control their internal structural order, or low microscopic entropy. The formal definition of this important concept of *'variable topology'* was introduced in our recent paper [33] in the context of the spacetime evolution of organisms, populations and species. Interestingly, for many multi-cellular organisms, including man, the overall morphological symmetry (but not the internal organizational topology) is retained from the beginning of ontogenetic development is externally bilateral–just one plane of mirror symmetry– from *Planaria* to humans. The presence of the head-to-tail asymmetry introduces increasingly marked differences among the various areas of the head, middle, or tail regions as the organism develops. There is however in man– as in other mammals– an internal bilateral asymmetry (e.g., only one heart on the left side), as well as a front to back, both external and internal anatomical asymmetry. In the case of the brain, however, only humans seem to have a significant bilateral, internal asymmetry between the two brain hemispheres that interestingly relates to the speech-related 'centers' (located in the majority of humans in the left brain hemisphere).

The multiplicity of boundaries, and the dynamics that derive from it, generate interesting phenomena. Boundaries tend to reinforce each other, as in the case of dissipative structures formed through coupled chemical, chaotic reactions. According to Poli in [210], *"this somewhat astonishing regularity of nature has not been sufficiently emphasized in perception philosophy."*

4.2. *Simple and Super–Complex Dynamics: Closed vs. Open Systems*

In an early report [11], Baianu and Marinescu considered the possibility of formulating a *Supercategorical Unitary Theory of Systems* (i.e., including both simple and complex ones, etc.) and pointed out several, possible representations of general systems and organisms both in terms of organizational structure and dynamics. Furthermore, it was proposed that the formulation of any model or 'simulation' of a complex system– such as living organism or a society–involves generating a first–stage *logical model* (not-necessarily Boolean!), followed by a *mathematical* one, *complete with structure* [18]. Then, it was pointed out that such a modelling process involves a diagram containing the complex system, (**CS**) and its dynamics, a corresponding, initial logical model, **L**, *'encoding'* the essential dynamic and/or structural properties of **CS**, and a detailed, structured mathematical model (**M**); this initial modelling diagram may or may not be commutative, and the modelling can be iterated through modifications of ***L***, and/or **M**, until an acceptable agreement is achieved between the behaviour of the model and that of the natural, complex system [11]. Such an *iterative modelling* process may ultimately 'converge' to appropriate models of the complex system, and perhaps a best possible model could be attained as the categorical colimit of the directed family of diagrams generated through such a modelling process. The possible models **L**, or especially **M**, were not considered to be necessarily either numerical or recursively computable (e.g., with an algorithm or a software program) by a digital computer [23],[34]. The mathematician John von Neumann regarded 'complexity' as a measurable property of natural systems below the threshold of which systems behave 'simply', but above which they evolve, reproduce, self–organize, etc. It was claimed that any 'natural' system fits this profile. But the classical assumption that natural systems are simple, or 'mechanistic', is too restrictive since 'simple' is applicable only to machines, closed physicochemical systems, computers, or any system which is *recursively computable.* Robert Rosen also proposed in 1987 a major refinement of these ideas about complexity by a more exact classification of 'simple' and 'complex' systems [232]. Thus, *simple systems* can be characterized through representations which admit *maximal* models, and can be therefore re–assimilated via a hierarchy of informational levels. Moreover, the duality between dynamical systems and states is also a characteristic of such simple dynamical systems. Furthermore, *complex systems* do not admit any maximal model. On the other hand, at the next, higher level of complexity, an *ultra-complex system*– as applied to psychological or sociological structures– can only be described in terms of *variable categories* or structures, and thus cannot be reasonably represented by a fixed state space for its entire lifespan. Simulations through limiting dynamical approximations lead to increasing system 'errors'. Just as for simple systems, both *super–complex* and *ultra-complex* systems admit their own orders of causation, but the latter two types are different from the first–by inclusion rather than exclusion– of the 'mechanisms' that control simple dynamical systems. Unfortunately, in the quite extensive literature on experimental biology, biomedicine and ecology, the term 'mechanism' has been widely employed instead of that of logical explanation of any natural process, thus assuming implicitly that either all systems are simple or that any complex dynamics can be derived from simple system dynamics, which is naturally incorrect.

4.3. *Commutative vs. Non-commutative Dynamic Modelling Diagrams*

Interestingly, Rosen also showed in 1987 that complex dynamical systems, such as biological organisms, cannot be adequately modelled through a *commutative* modelling diagram [232] – in the sense of digital computer simulation–whereas the simple ('physical'/ engineering) dynamical systems can be thus numerically simulated. Furthermore, his modelling commutative diagram for a *simple dynamical system* included both the 'encoding' of the 'real' system **N** in (**M**) as well as the 'decoding' of (**M**) back into **N**:

$$\begin{array}{ccc} [SYSTEM] & \xrightarrow{\textbf{Encoding...}\hookrightarrow} & LOGICS \oplus MATHS. \\ \delta \Big\downarrow & & \Big\downarrow \aleph_M \\ SYSTEM & \xleftarrow[\textbf{Decoding} \hookleftarrow \ldots]{} & [MATHS.\square MODEL] \end{array}$$

where δ is the real system dynamics and $\aleph$ is an algorithm implementing the numerical computation of the mathematical model (**M**) on a digital computer. Firstly, one notes the ominous absence of the *Logical Model*, ***L***, from Rosen's diagram published in 1987. Secondly, one also notes the obvious presence of logical arguments and indeed (*non-Boolean*) 'schemes' related to the entailment of organismic models, such as **MR**-systems, in the more recent books that were published last by Robert Rosen in 1997 [237] and 2000 [238]. This aspect will be further discussed in Section 4, with the full mathematical details provided in the paper by Brown, Glazebrook and Baianu [69]. Furthermore, Elsasser pointed out in 1980 a fundamental, logical difference between physical systems and biosystems or organisms: whereas the former are readily represented by *homogeneous* logic classes, living organisms exhibit considerable variability and can only be represented by *heterogeneous* logic classes [107]. One can readily represent homogeneous logic classes or endow them with 'uniform' mathematical structures, but heterogeneous ones are far more elusive and may admit a multiplicity of mathematical representations or possess variable structure. This logical criterion may thus be useful for further distinguishing simple systems from highly complex systems.

The importance of *Logic Algebras*, and indeed of *Categories of Logic Algebras*, is rarely discussed in modern Ontology even though categorical formulations of specific Ontology domains such as Biological Ontology and Neural Network Ontology are being extensively developed. For a recent review of such categories of logic algebras the reader is referred to the concise presentation by Georgescu in 2006 [120]; their relevance to network biodynamics was also recently assessed [30] –[35].

Super-complex systems, such as those supporting neurophysiological activities, are explained only in terms of non–linear, rather than linear causality. In some way then, these systems are not normally considered as part of either traditional physics or the complex 'chaotic' systems physics that are known to be fully deterministic. However, super-complex (biological) systems have the potential to manifest novel and counter–intuitive behavior such as in the manifestation of 'emergence', ontogenetic development, morphogenesis and biological evolution. (The precise meaning of supercomplex systems is also formally defined in the sequel).

4.4. *Comparing Systems: Similarity Relations between Analogous or Adjoint Systems. Diagrams Linking Super– and Ultra– Complex Meta–Levels: Classification as a Dynamic Analogy. Categorical Adjointness as Functional Homology*

Categorical-based comparisons of different types of systems in diagrams provide useful means for their classification and understanding the relations between them. From a global viewpoint, comparing categories of such different systems does reveal useful analogies, or similarities, between systems and also their universal properties. According to Rashevsky in [224], general relations between sets of biological organisms can be compared with those between societies, thus leading to more general principles pertaining to both. This can be considered as a further, practically useful elaboration of Spencer's philosophical principle ideas in biology and sociology. When viewed from a formal perspective of Poli's theory of levels, as further developed by Baianu and Poli in this volume [40], the two levels of super– and ultra– complex systems are quite *distinct* in many of their defining properties, and therefore, categorical diagrams that 'mix' such distinct levels *do not commute*.

Considering dynamic similarity, Robert Rosen introduced in 1968 the concept of *'analogous'* (classical) dynamical systems in terms of categorical, dynamic isomorphisms between their isomorphic state-spaces that commute with their transition (state) function, or dynamic laws [234]. However, the extension of this concept to either complex or super-complex systems has not yet been investigated, and may be similar in importance to the introduction of the Lorentz-Poincaré group of transformations for reference frames in Relativity theory. On the other hand, one is often looking for *relational invariance* or *similarity in functionality* between different organisms or between different stages of development during ontogeny–the development of an organism from a fertilized egg. In this context, the categorical concept of '*dynamically adjoint systems*' was introduced in relation to the data obtained through nuclear transplant experiments [15].

D4. *Dynamically Adjoint, General Systems.* Then, extending the latter concept to super– and ultra–complex systems , one has in general, that two complex, super–complex or ultra-complex systems with 'state spaces' being defined respectively as A and A^*, are *dynamically adjoint* if they can be represented naturally by the following (functorial) diagram:

$$\begin{array}{ccc} A & \xrightarrow{F} & A^* \\ {\scriptstyle F'}\downarrow & & \downarrow{\scriptstyle G} \\ A^* & \xrightarrow[G']{} & A \end{array} \tag{0.1}$$

with $F \approx F'$ and $G \approx G'$ being isomorphic (that is, $\approx$ representing natural equivalences between adjoint functors of the same kind, either left or right), and as above in diagram (0.1), the two diagonals are, respectively, the state-space transition functions $\Delta : A \to A$ and $\Delta^* : A^* \to A^*$ of the two adjoint dynamical systems. (It would also be interesting to investigate dynamic adjointness in the context of quantum dynamical systems and quantum automata, as defined in [13]-[14], [39]).

A *left-adjoint* functor, such as the functor F in the above commutative diagram between categories representing state spaces of equivalent cell nuclei *preserves limits*, whereas the *right-adjoint* (or coadjoint) functor, such as G above, *preserves colimits.* (For precise definitions of adjoint functors the reader is referred to [15], [213] , [256], and the initial paper by Kan in 1958 [154]).

Thus, dynamical attractors and genericity of states are preserved for differentiating cells up to the blastula stage of organismic development. Subsequent stages of ontogenetic development can only be considered as 'weakly adjoint' , or partially analogous. Cloned, higher organisms are further examples of weak adjointess between closely related organisms. Similar dynamic controls may operate for controlling division cycles in the cells of different organisms; therefore, such instances are also good example of the dynamic adjointness relation (or, more generally, of weak adjointness relations) between cell types of different organisms that may be very far apart phylogenetically, and even located on different 'branches of the tree of life.' A more elaborate dynamic concept of 'relational homology' between the genomes of different species during evolution was also proposed by Baianu in [21], suggesting that an entire phylogenetic series can be characterized by a topologically–rather than biologically–*homologous sequence* of genomes which preserves certain groups of genes encoding *the essential* biological functions. A striking example was recently suggested involving the differentiation of the nervous system in the fruit fly and mice (and perhaps also man) which leads to the formation of the back, middle and front parts of the neural tube. A related, topological generalization of such a dynamic similarity between systems was previously introduced as *topological conjugacy*, developed by Baianu in [23], and by Baianu and Lin in [28], which replaces the recursive, digital simulation with *symbolic- topological modelling* for both super– and ultra– complex systems [32]–[34].

This approach stems logically from the introduction of topological/symbolic computation and topological computers [14], [23],[28]), as well as their natural extensions to quantum nano-automata [29], quantum automata and quantum computers ([13]-[14] and [25], [31], respectively); the latter may allow us to make a 'quantum leap' in our understanding Life and the higher complexity levels in general. Such is also the relevance of Quantum Logics and LM-logic algebra to understand the immanent operational logics of the human brain and the associated mind meta–level. Quantum Logics concepts are introduced next that are also relevant to the fundamental, or 'ultimate', concept of spacetime, well-beyond our phenomenal reach, and thus in this specific sense, *transcedental* to our physical experience (perhaps vindicating the need for a Kantian–like *transcedental logic*, but from a quite different standpoint than that originally advanced by Kant in his critique of 'pure' reason; instead of being 'mystical'- as Husserl might have said–the transcendental logic of quantized spacetime is very different from the Boolean logic of digital computers, as it is *quantum*, and thus non–commutative). A *Transcendental Ontology*, whereas with a definite Kantian 'flavor', would not be at all 'mystical' in Husserl's sense, but would rely on 'verifiable' many–valued, non–commutative logics, and thus contrary to Kant's original presupposition [155], as well as untouchable by Husserl's critique. The fundamental nature of spacetime would be 'provable' and 'verifiable', but only to the extent allowed by Quantum Logics, not by an arbitrary ('mystical') Kantian–transcendental logic or by impossible, direct phenomenal observations at the Planck scale.

4.5. *Fundamental Concepts of Algebraic Topology with Potential Application to Ontology Levels Theory and the Classification of SpaceTime Structures*

We shall consider in this section the potential impact of novel Algebraic Topology concepts, methods and results on the problems of defining and classifying rigorously Quantum Spacetimes (QSS)[3], [36]-[38],[69], [78]-[79].

Traditional algebraic topology works by several methods, but all involve going from a space to some form of combinatorial or algebraic structure. The earliest of these methods was 'triangulation': a space was supposed resented as a simplicial complex, i.e. was subdivided into simplices of various dimensions glued together along faces, and an algebraic structure such as a chain complex was built out of this simplicial complex, once assigned an orientation, or, as found convenient later, a total order on the vertices. Then in the 1940s a convenient form of singular theory was found, which assigned to any space X a 'singular simplicial set' SX, using continuous mappings from $\Delta^n \to X$, where Δ^n is the standard n-simplex. From this simplicial set, the whole of the weak homotopy type could in principle be determined. An alternative approach was found by Čech, using an open covers $\mathcal{U}$ of X to determine a simplicial set $N\mathcal{U}$, and then refining the covers to get better 'approximations' to X. It was this method which Grothendieck discovered could be extended, especially combined with new methods of homological algebra, and the theory of sheaves, to give new applications of algebraic topology to algebraic geometry, via his theory of schemes. The 600-page project manuscript, *'Pursuing Stacks'* written by Alexander Grothendieck in 1983 was partly aimed at a *non-Abelian homological algebra*; it did not achieve this goal but has been very influential in the development of weak n-categories and other *higher categorical structures* that are relevant to QSS structures. With the advent of Quantum Groupoids–generalizing quantum groups, Quantum Algebra and Quantum Algebraic Topology, several fundamental concepts and new theorems of Algebraic Topology may also acquire an increased importance through their potential applications to current problems in theoretical and mathematical physics, such as those described in an available preprint [38], and also in several other recent publications [36]–[37], [69]. If quantum theory is to reject the notion of a continuum, then it must also reject the notion of the real line and the notion of a path. How then is one to construct a homotopy theory?

One possibility is to take the route signalled by Čech, and which later developed in the hands of Borsuk into 'Shape Theory' (see, Cordier and Porter, 1989). Thus a quite general space is studied by means of its approximation by open covers. Yet another possible approach is briefly pointed out in the next subsection.

In such novel applications, both the internal and external groupoid symmetries [265] may too acquire new physical significance. Thus, if quantum theories were to reject the notion of a *continuum* model for spacetime, then it would also have to reject the notion of the real line and the notion of a path. How then is one to construct a homotopy theory? One possibility is to take the route signalled by Čech [82], and which later developed in the hands of Borsuk into 'Shape Theory' [86]. Thus a quite general space is studied indirectly by means of its approximation by open covers. Yet another possible approach is briefly outlined in the next section.

Several fundamental concepts of Algebraic Topology and Category Theory that are needed throughout this monograph will be introduced next so that we can reach an extremely wide range of applicability, especially to the higher complexity levels of reality. Full mathematical details are available in a recent paper by Brown et al. [69] that focused on a mathematical–conceptual framework for a formal approach to Categorical Ontology and the Theory of Ontological Levels [206], [40], and also in the **Appendix**.

4.5.1. *Groupoids, Topological Groupoids and Groupoid Atlases*

D5. Recall that a *groupoid* G is a small category in which every morphism is an isomorphism.

Topological Groupoids

D6. An especially interesting concept is that of a *topological groupoid* which is a groupoid internal to the category Top; further mathematical details are presented in the paper by Brown et al. in 2007 [69], and also in the **Appendix**.

Groupoid Atlases

Motivation for the notion of a groupoid atlas comes from considering families of group actions, in the first instance on the same set. As a notable instance, a subgroup H of a group G gives rise to a group action of H on G whose orbits are the cosets of H in G. However a common situation is to have more than one subgroup of G, and then the various actions of these subgroups on G are related to the actions of the intersections of the subgroups. This situation is handled by the notion of *global action*, as defined in [41]. A key point in this construction is that the orbits of a group action then become the connected components of a groupoid. Also this enables relations with other uses of groupoids. The above account motivates the following.

D7. A *groupoid atlas* $\mathcal{A}$ on a set $X_{\mathcal{A}}$ consists of a family of 'local groupoids' $(\mathsf{G}_{\mathcal{A}})$ defined with respective object sets $(X_{\mathcal{A}})_\alpha$ taken to be subsets of $X_{\mathcal{A}}$. These local groupoids are indexed by a set $\Psi_{\mathcal{A}}$, again called the *coordinate system of* $\mathcal{A}$ which is equipped with a reflexive relation denoted by $\leqslant$. This data is to satisfy several conditions reported in [41] by Bak et al. in 2006, and also discussed in [63] in the context of Categorical Ontology.

4.5.2. *The van Kampen Theorem and its Generalisations to Groupoids and Higher Homotopy*

The van Kampen Theorem has an important and also anomalous rôle in algebraic topology. It allows computation of an important invariant for spaces built up out of simpler ones. It is anomalous because it deals with a non-Abelian invariant, and has not generally been seen as having higher dimensional analogues. However, Brown found in 1967 a generalisation of this theorem to the *fundamental groupoid* $\pi_1(X, X_0)$ of X on a set X_0 of base points [60,63]. Such methods were extended successfully by Brown to *higher dimensions*, [64],[68]. The potential applications of the Higher Homotopy van Kampen Theorems [36] were already discussed in a previous paper [69] published by Brown, Glazebrook and Baianu in 2007, and are further detailed in the Appendix.

5. Complex Systems Biology. Emergence of Life and Evolutionary Biology

5.1. *Towards Biological Postulates and Principles*

Whereas the hierarchical theory of levels provides a powerful, systems approach through categorical ontology, the foundation of science involves *universal* models and theories pertaining to different levels of reality. It would seem natural to expect that theories aimed at different ontological levels of reality should have different principles. We are advocating the need for developing precise, but nevertheless 'flexible', concepts and novel mathematical representations suitable for understanding the emergence of the higher complexity levels of reality. Such theories are based on axioms, principles, postulates and laws operating on distinct levels of reality with a specific degree of complexity. Because of such distinctions, inter-level principles or laws are rare and over-simplified principles abound. Alternative approaches may be, however, possible based upon an improved ontological theory of levels. Interestingly, the founder of Relational Biology, Nicolas Rashevsky proposed in 1969 that physical laws and principles can be expressed in terms of *mathematical functions*, or mappings, and are thus being predominantly expressed in a *numerical* form, whereas the laws and principles of biological organisms and societies need take a more general form in terms of quite general, or abstract–mathematical and logical relations which cannot always be expressed numerically; the latter are often qualitative, whereas the former are predominantly quantitative [224].

Rashevsky focused his Relational Biology/Society Organization papers on a search for more general relations in Biology and Sociology that are also compatible with the former. Furthermore, Rashevsky proposed two biological principles that add to Darwin's natural selection and the 'survival of the fittest principle', *the emergent relational structure that are defining the adaptive organism*:

1. The Principle of Optimal Design[233],
and
2. The Principle of Relational Invariance (initially phrased by Rashevsky as *"Biological Epimorphism"*)[12]-[13],[15],[222].

In essence, the 'Principle of Optimal Design' [233] defines the organization and structure of the 'fittest' organism which survives in the natural selection process of competition between species, in terms of an extremal criterion, similar to that of Maupertuis; the optimally 'designed' organism is that which acquires maximum functionality essential to survival of the successful species at the lowest 'cost' possible [11]-[13]. The 'design' in this case is commonly taken in the sense of the result of a long evolutionary process that occurred under various environmental and propagation constraints or selection 'pressures', such as that caused by sexual reproduction in Darwin's model of the origin of species during biological evolution. The 'costs' are here defined in the context of the environmental niche in terms of material, energy, genetic and organismic processes required to produce/entail the pre-requisite biological function(s) and their supporting anatomical structure(s) needed for competitive survival in the selected niche. Further details were presented by Robert Rosen in his short, but significant, book on optimality principles in theoretical biology [233], published in 1967.

The 'Principle of Biological Epimorphism', on the other hand, states that the highly specialized biological functions of higher organisms can be mapped (through an epimorphism) onto those of the simpler organisms, and ultimately onto those of a (hypothetical) primordial organism (which is assumed to be unique up to an isomorphism or *selection-equivalence*). The latter proposition, as formulated by Rashevsky, is more akin to a postulate than a principle. However, it was then generalised and re-stated as the Postulate of Relational Invariance [12]. Somewhat similarly, a dual principle and the colimit construction were invoked for the ontogenetic development of organisms [11], and more recently other quite similar colimit constructions were considered in relation to 'Memory Evolutive Systems', or phylogeny [103]-[104].

An axiomatic system (ETAS) leading to higher dimensional algebras of organisms in supercategories has also been formulated [18] which specifies both the logical and the mathematical ($\pi-$) structures required for complete self-reproduction and self-reference, self-awareness, etc. of living organisms. To date, there is no higher dimensional algebra (HDA) axiomatics other than the ETAS proposed for complete self-reproduction in super-complex systems, or for self-reference in ultra-complex ones. On the other hand, the preceding, simpler ETAC axiomatics introduced by Lawvere, was proposed for the foundation of 'all' mathematics, including categories [166]-[167], but this seems to have occurred before the actual emergence of HDA.

5.2. *Super-Complex System Dynamics in Living Organisms: Genericity, Multi-Stability and Variable State Spaces*

The important claim is here defended that above the level of 'complex systems with chaos' there is still present a higher, super-complexity level of living organisms –which are neither machines/simple dynamical systems nor are they mere 'chaotically'– behaving systems, in the sense usually employed by the physical theory of 'chaotic' dynamics. These distinct levels, physical/chaotic and biological were represented as distinct objects in the non-commutative diagram of the previous section joined by causal links, running from simple to 'chaotic–complex' (physical) dynamics, then upwards linked to super-complex biodynamics, and still higher to the ultra-complex, meta-level of mental dynamic processes of processes. A further claim is defended that even though the higher levels are linked to– and indeed subsume, or include – the lower ones in their distinct organization, they are not reducible in a physical or (bio) chemical sense to the lower dynamic level. *In esse*, the distinction between the existence of the higher, super– and ultra– complexity levels and the physical/chemical level of reality can only be made on the basis of their dynamics. Neither Life nor the Mind can be properly conceived as *static*/closed systems, or even as quasi–static structures, without either a time-dependence or associated, material (including energy) and microentropy/gradient-driven flows which are characteristic of *irreversible, open* systems. If the super-complex dynamics stops so does life. Somewhat similarly, but at a different, higher level of reality, the human mind's ultra–complex existence emerges as a dynamic meta-process of processes, supported also by neural dynamics and life. Artificially separating the mind from the human brain and life in an abstract–'analytical' sense, as in Cartesian Dualism, promotes a static view and an object–based approach that might be relevant, or just partially applicable only to *unconscious* human beings, such as in the case of a severe brain stroke, or even worse, in cases caused by permanent, irrecoverable human brain injuries to a 'living–vegetable' status in grave, major accidents.

In a logical context, biological organisms were also shown to be *extremely* complex [107]); this implies that any well-founded theoretical biology requires an unique axiomatics [12]. We proposed that all functional (or living) organisms also exhibit super-complex dynamics [13]-[15], and therefore that their theoretical understanding also requires new biological or relational principles [11]-[13],[15],[17]–[18],[220]-[224],[233]. The *non-computability* of the biodynamics of functional organisms with recursive functions, digital computers or Boolean algorithms [23],[28],[229],[232],[237]-[238] is a major obstacle to quantitative approaches in biological studies, as well as a major theoretical stumbling block for computer scientists and biomedical researchers who attempt to model biological systems [23],[28] without taking into account the limitations of computer simulations of biodynamics of whole organisms or entire organs.

In the next sections we shall examine in further detail how super-complex dynamics emerges in organisms from the *molecular and supra-molecular* levels that recently have already been claimed to exist by several experimental molecular biologists in the form of 'super-complex' molecular assemblies. As shown in previous reports [11]-[13],[19],[32], multicellular organismal development, or ontogeny, can be represented as a directed system or family of dynamic state spaces corresponding to all stages of ontogenetic development of increasing dimensionality. The *colimit* of this *directed system* of ontogenetic stages/dynamic state spaces represents the *mature* stage of the organism [12],[15],[32]. This emergent process involved in ontogeny leads directly to *variable*, super-complex dynamics and *higher dimensional* state spaces. As an over-simplified, pictorial–but also formalizable– representation, let us consider a living cell as a topological 'cell' or simplex of a CW-complex. Then, as a multicellular organism develops a complete simplicial (CW) complex emerges as an over-simplified picture of the whole, mature organism. The higher dimensionality then emerges by considering each cell with its associated, *variable* dynamic state space [12],[19],[22]. As shown in previous reports [11]-[13], the corresponding variable dynamic structure representing biological relations, functionalities and dynamic transitions is an organismic supercategory, or **OS**. The time-ordered sequence of CW-complexes of increasing dimensionality associated with the development of a multi-cellular organism provides a specific example of a *variable topology.* The 'boundary conditions' or constraints imposed by the environment on the organismic development will then lead to context-dependent variable topologies that are not strictly determined by the genome or developing genetic networks. Although ontogenetic development is usually structurally stable there exist teratogenic conditions or agents that can 'de-stabilize' the developing organism, thus leading to abnormal development. One also has the possibility of abnormal organismic, or brain, development caused by altered genomes, as for example in those cases of autism caused by the fragile-X chromosome syndrome. On the other hand, both single-cell and multi-cellular organisms can be represented in terms of variable dynamic systems, such as generalised **(M,R)**- systems [16]-[17], including dynamic realizations of **(M,R)**- systems [23], [235]-[236].

5.2.1. *Organisms Represented as Variable Dynamic Systems: Generic States and Dynamic System Genericity.*

In actual fact, the super-complexity of the organism itself emerges through the generation of dynamic, variable structures which then also entail variable/flexible functions, homeostasis, autopoiesis, anticipation, and so on. In this context, it is interesting that Wiener in [269] proposed the simulation of living organisms by variable machines/automata that did not exist in his time. The latter were subsequently formalised independently in two related reports [22],[24],[27] .

The topologist René Thom's metaphorical approach of Catastrophe Theory [258] to biodynamics, provides some insights into *structural stability* and biodynamics *via* 'generic' states that when perturbed lead to other similarly stable states. When viewed from a categorical standpoint, organismic dynamics has been suggested to be characterized not only by homeostatic processes and steady state, but also by *multi-stability* [5],[12],[18]. The latter concept is clearly equivalent from a dynamic/topological standpoint to super-complex system genericity, and the presence of *multiple dynamic attractors* [13] which were categorically represented as *commutative super–pushouts* [12],[15]. The presence of generic states and regions in super-complex system dynamics is thus linked to the emergence of complexity through both structural stability and the *open* system attribute of any living organism that enable its persistence in time, in an accommodating niche, suitable for its competitive survival.

5.3. *Anticipation in Super– and Ultra– complex Systems: Feedbacks and Feedforward. Autopoiesis*

Rosen in [229],[232],[236] characterised a change of state as governed by a predicted future state of the organism and/or in respect of its environment. These factors appear separately from the idea of simple systems since future influence (*via* inputs, etc.) are not seen as compatible with classical, deterministic causality of classical mechanical systems or even classical statistical mechanics. Any effort to monitor a complex system through a predictive dynamic model results in a growing discrepancy between the actual function of the system and its predicative counterpart thus leading to a (global) system failure [232]. Furthermore, *anticipatory behaviour*, considered apart from any non–feedback mechanism, is realized in all levels of biological organization, or the broad–scale *autopoiesis* of structurally linked systems/processes that continually inter-adjust with their environment over time [173],[182]. Within a social system the autopoiesis of the various components is a necessary and sufficient condition for the realisation of the system itself. In this respect, the structure of a society as a particular instance of a social system is determined by the structural framework of the (autopoietic components) and the sum total of collective interactive relations. Consequently, the societal framework is based upon a selection of its component structures in providing a medium in which these components realize their ontogeny. It is just through participation alone that an autopoietic system determines a social system by realizing the relations that are characteristic of that system. Then, the huge number and variety of biological organisms formed through evolution can be understood as a result of the very numerous combinatorial potentialities of *super-complex* systems, as well as the large number of different environmental factors available to organismic evolution.

5.4. *Ultra-Complex Systems: The Emergence of the Unique Ultra-Complexity through Co-Evolution of the Human Mind and Society.*

Higher still than the organismic level characterized by super-complex dynamics, there emerged perhaps even earlier than 400,000 years ago the *unique, ultra-complex* levels of human mind/consciousness and human society interactions, as it will be further discussed in the following sections. There is now only one species known who is capable of rational, symbolic/abstract and creative thinking as part-and-parcel of consciousness–*Homo sapiens sapiens*– which seems to have descended from a common ancestor with *Homo ergaster*, and separated from the latter some 2.2 million years ago. However, the oldest fossils of *H. sapiens* found so far are just about 400,000 years old.

The following diagram summarizes the relationships/links between such different systems on different ontological levels of increasing complexity from the simple dynamics of physical systems to the ultra-complex, global dynamics of psychological processes, collectively known as 'human consciousness'. With the emergence of the ultra-complex system of the human mind– based on the super-complex human organism– there is always an associated progression towards higher dimensional algebras from the lower dimensions of human neural network dynamics and the simple algebra of physical dynamics, as shown in the following, essentially *non-commutative* categorical ontology diagram. This is similar–but not isomorphic– to the higher dimensionality emergence that occurs during ontogenetic development of an organism, as discussed in the previous subsection.

$$\begin{array}{ccc}
[SUPER-COMPLEX] & \xrightarrow{\textbf{(Higher Dim)}} & ULTRA-COMPLEX \\
\Big\downarrow \Lambda & & \Big\downarrow onto \\
COMPLEX & \xleftarrow[\textbf{(Generic Map)}]{} & [SIMPLE]
\end{array}$$

Note that the above diagram is indeed not 'natural' for reasons related to the emergent higher dimensions of the super–complex (biological/organismic) and/or ultra–complex (psychological/neural network dynamic) levels in comparison with the low dimensions of either simple (physical/classical) or complex (chaotic) dynamic systems. It might be possible, at least in principle, to obtain commutativity by replacing the simple dynamical system in the diagram with a quantum system, or a quantum 'automaton' [13]-[14],[25]); however, in this case the diagram still does not necessarily close between the quantum system and the complex system with chaos, because it would seem that *quantum systems are 'fuzzy'*–not strictly deterministic– as complex 'chaotic' systems are. Furthermore, this categorical ontology diagram is neither recursively computable nor representable through a commutative algorithm of the kind proposed for Boolean neural networks [187]; for an extensive review of network biodynamic modelling, 'simulations' and also non-computability issues for biological systems see ref. [23],[28], and references cited therein. Note also that the top layer of the diagram has generic states and generic regions, whereas the lower layer does not; the top layer lives, the bottom one does not.

5.5. *Connectivity and Bionetwork Topology: Genetic Ontology and Interactomics Reconstruction.*

One may place special emphasis on network topology and connectivity in Genetic Ontology and Categorical Biology since these concepts are becoming increasingly important in modern biology, as realized in rapidly unfolding areas such as post-Genomic Biology, *Proteomics* and *Interactomics* that aim at relating structure and protein-protein-biomolecule interactions to biological function. The categories of the biological/genetic/ecological/ levels may be seen as more 'structured' compared with those of the cognitive/mental levels (hinging on epiphenomenalism, interactive dualism, etc.) which may be seen as 'less structured', but not necessarily having less structural power owing to the increased abstraction in their design of representation. We are here somewhat in concert with Hartmann's *laws of autonomy* [137].

5.6. *The Emergence of Life*

With an increasing level of complexity generated through billions of years of evolution in the beginning, followed by millions of years for the ascent of man, and perhaps about 10,000 more years for human societies and their civilizations, there is an increasing degree of *genericity* for the dynamic states of the evolving systems [232],[258]. If such genericity is sufficient for the survival of the relatively young human civilisation–by comparison with the total length of evolution of about 2 billion years– it is arguably one of *the most important human ontology questions*. Evolutionary theories based only on historical evidence, and also without a dynamic foundation, cannot obviously answer this ardent question.

5.6.1. *What is Life ?*

Although the distinction between living organisms and simple physical systems, machines, robots and computer simulations appears obvious at first sight, the profound differences that exist both in terms of dynamics, construction and structure require a great deal of thought, conceptual analysis, development and integration or synthesis. This fundamental, ontological question about Life occurs in various forms, possibly with quite different attempts at answers, in several books (e.g., Schrödinger's [242] and the last two books by Rosen [237]-[238]).

5.6.2. *Emergence of Super-Complex Systems and Life. The 'Primordial' as the Simplest (M,R)- or Autopoietic- System*

In the preceding two sections we have already discussed from the categorical viewpoint several key systemic differences in terms of dynamics and modelling between living and inanimate systems. The ontology of super-complex biological systems, or biosystems (BIS), has perhaps begun with Elsasser's paper [107] who recognized that organisms are extremely complex systems, that they exhibit wide variability in behaviour and dynamics, and also from a logical viewpoint, that they form– unlike physical systems– *heterogeneous classes*. (We shall use the 'shorthand' term *'biosystems'* to stand for super-complex biological systems, thus implicitly specifying the attribute super-complex within biosystems). This intrinsic BIS variability was previously recognized as *fuzziness* [11],[20] and some of its possible origins were suggested to be found in the partial structural disorder of biopolymers and biomembranes [20]. Still other basic reasons for the presence of both dynamic and structural *'bio-fuzziness'* is the 'immanent' LM-logic in biosystems, such as functional genetic networks, and possibly also the Q-logic of signalling pathways in living cells. There are, however, significant differences between Quantum Logic, which is also non-commutative, and the LM-Logics of Life processes. Whereas certain reductionists would attempt to reduce Life's logics, or even human consciousness, to Quantum Logic (QL), the former are at least logically and algebraically *not reducible to QL*. Nonetheless, it may be possible to formulate QL through certain modifications of *non-commutative LM-logics* [32]-[38].

Perhaps the most important attributes of Life are those related to the logics 'immanent' in those processes that are essential to Life. As an example, the logics and logic-algebras associated with functioning neuronal networks in the human brain–which are different from the multi–valued (Łukasiewicz–Moisil) logics [120] associated with functional genetic networks [18],[23],[28],[32] and self-reproduction [12],[18],[24],[171]. were shown to be different from the simple Boolean-crysippian logic upon which machines and computers are built by humans. The former n-valued (LM) logics of functional neuronal or genetic networks are *non-commutative* ones, leading to *non-linear, super-complex* dynamics, whereas the simple logics of 'physical' dynamic systems and machines/automata are *commutative* (in the sense of involving a commutative lattice structure). Here, we find a fundamental, logical reason why living organisms are *non-commutative*, super-complex systems, whereas simple dynamical systems have *commutative modelling diagrams* that are based on *commutative Boolean* logic. We also have here the reason why a *commutative* Categorical Ontology of Neural networks leads to advanced robotics and AI, but has indeed little to do with the *'immanent logics'* and functioning of the living brain, contrary to the proposition made by McCulloch and Pitts in 1943 [187].

There have been several attempts at defining life in reductionistic terms and a few non-reductionist ones. Rashevsky attempted in 1967 to define life in terms of the essential functional relations arising between organismic sets of various orders, i.e. ontological levels, beginning with genetic sets, their activities and products as the lowest possible order, zero, of on 'organismic set' (OS) [222]. Previously, he considered representations of biological activities in terms of logical Boolean predicates [221], undoubtedly influenced by the earlier work of McCulloch and Pitts [187]. He attempted to provide the simplest model possible and he proposed in [222] the organismic set, or OS, as a basic representation of living systems, but he did not attempt himself to endow his OS with either a topological or categorical structure, in spite of the fact that he previously reported on the fundamental connection between Topology and Life [220]. He did attempt, however, a logical analysis in terms of formal symbolic logics and Hilbert's predicates. Furthermore, his PhD student, Robert Rosen did take up the challenge of representing organisms in terms of simple categorical models– his Metabolic-Repair,(**M,R**)-systems, or (**MR**)s [230]. His two seminal papers were then followed by a series of follow up reports with many interesting, biologically relevant results and consequences in spite of the simplicity of the MR, categorical set 'structure'. Further extensions and generalisations of MR's were subsequently explored by considering abstract categories with both algebraic and topological structures [16]-[17],[23],[235]-[236],[264].

On the one hand, simple dynamical (physical) systems are often represented through groups of dynamic transformations. In GR, for example, these would be Lorentz–Poincare' groups of spacetime transformations/reference frames. On the other hand, super-complex systems, or biosystems, emerging through self-organization and complex aggregation of simple dynamical ones, are therefore expected to be represented mathematically–at least on the next level of complexity– through an extension, or generalisations of mathematical groups, such as, for example, *groupoids.* Whereas simple physical systems with linear causality have high symmetry, a single energy minimum, and thus they possess only *degenerate* dynamics, the super-complex (living) systems emerge with lower symmetries but higher dynamic and functional/relational complexity. As symmetries get 'broken' the complexity degree increases sharply. From groups that can be considered as very simple categories that have just one object and reversible/invertible endomorphisms, one moves through 'symmetry breaking' to the structurally more complex groupoids, that are categories with many objects but still with all morphisms invertible. Dynamically, this reflects the transition from degenerate dynamics with one, or a few stable, isolated states ('degenerate' ones) to dynamic state regions of many generic states that are metastable; this multi-stability of biodynamics is nicely captured by the many objects of the groupoid and is the key to the 'flow of life' occurring as multiple transitions between the multiple metastable states of the homeostatic, living system. More details of how the latter emerge through biomolecular reactions, such as catabolic/anabolic reactions, will be presented in the next subsections, and also in the next section, especially under natural transformations of functors of biomolecular categories. As we shall see in later sections, the emergence of human consciousness as an ultra-complex process became possible through the development of the *bilaterally asymmetric* human brain, not just through a mere increase in size, but a basic change in brain architecture as well. Relationally, this is reflected in the transition to a higher dimensional structure, for example a double biogroupoid representing the bilaterally asymmetric human brain architecture and functions, as we shall discuss further in this section. Therefore, we shall consider throughout the following sections various groupoids as some of the 'simplest' illustrations of the mathematical structures present in super-complex biological systems and classes thereof, such as *biogroupoids*, that is the groupoids that arise from the *equivalence classes* of either functional cells or organisms. In the case of classes of organisms that are equivalent from the viewpoint of reproduction such biogroupoids at the ecosystem level represent biological species. Then, one can represent speciation and evolving biological species as variable biogroupoids. Relevant here are also *crossed complexes* [64] of variable groupoids and/or *multi–groupoids* as more complex representations of biosystems that follow the emergence of ultra–complex systems (the mind and human societies, for example) from super-complex dynamic systems (organisms) [40],[69].

Furthermore, simple dynamic systems, or general automata, have *canonically decomposable semigroup* state spaces (the Krone-Rhodes Decomposition Theorem, cited in [23]). It is in this sense also that recursively computable systems are 'simple', whereas organisms are not. In contrast, super-complex systems do not have state spaces that are known to be canonically decomposable, or partitioned into functionally independent subcomponent spaces, that is within a living organism all organs are inter-dependent and integrated; one cannot generally find a subsystem or organ which retains organismic life–the full functionality of the whole organism. However, in some of the simpler organisms, for example in *Planaria*, regeneration of the whole organism is possible from several of its major parts. We note here that an interesting, incomplete but computable, model of multi-cellular organisms was formulated in terms of 'cellular' or 'tessellation' automata simulating cellular growth in planar arrays with such ideas leading and contributing towards the 'mirror neuron system hypothesis' [200]. Arbib's incomplete model of 'tessellation automata' is often used in one form or another by seekers of computer-generated/algorithmic, artificial 'life'.

5.6.3. *Emergence of Organisms, Essential Organismic Functions and Life. The Primordial.*

Whereas it would be desirable to have a complete definition of living organisms, the list of attributes needed for such a definition can be quite lengthy. In addition to super-complex, recursively non-computable and open system, there are several attributes employed to define living organisms, such as: auto-catalytic, self-organizing, structurally stable/generic, self-repair, self-reproducing, autopoietic, anticipatory, multi-level, and also possessing multi-valued logic. One needs to add to this list a number of processes that are thought to define life: irreversible processes coupled to bioenergetic processes and (bio)chemical concentration gradients, dissipative processes, inter-cellular flows, fluxes selectively mediated by semi-permeable biomembranes and thermodynamic linkage. These are of course just short lists that might be further condensed to a few key attributes and processes. However, some of these important attributes of organisms are inter-dependent and serve to define life categorically as a super-complex dynamic process that can have several alternate, or complementary descriptions/representations. Such descriptions can be formulated, for example, in terms of variable categories, variable groupoids, generalized Metabolic-Repair systems, organismic sets, hypergraphs, memory evolutive systems (MES), organismic toposes, interactomes, organismic super-categories and higher dimensional algebra. Each representation provides at present only a partial description of an organism, be it uni– or multi– cellular.

Organisms are thought of having all evolved from a simpler, 'primordial', proto-system or cell formed (how?) three, or perhaps four, billion years ago. Such a system, if considered to be the simplest, must have been similar to a bacterium, though perhaps without a cell wall, and also perhaps with a much smaller, single chromosome containing very few RNA 'genes' (two or, most likely, four).

We consider here a simple 'metaphor' of metabolic, self-repairing and self-reproducing models called (M,R)-systems, introduced by Robert Rosen [230]. Such models can represent some of the organismic functions that are essential to life; these models have been extensively studied and they can be further extended or generalized in several interesting ways. Rosen's simplest MR predicts one RNA 'gene' and just one proto-enzyme for the primordial 'organism'. An extended **MR** [16]-[17] predicts however the primordial, PMR, equipped with a *ribozyme* (a telomerase-like, proto-enzyme), and this PMR is then also capable of ribozyme- catalized DNA synthesis, and would have been perhaps surrounded by a 'simple' lipid-bilayer membrane some four billion years ago. This can be represented by the following, very simple diagram:

$$A \xrightarrow{f} B \xrightarrow{\Phi} \Re[\mathsf{A},\mathsf{B}] \xrightarrow{\beta} \Re[\mathsf{B},\Re[\mathsf{A},\mathsf{B}]] \xrightarrow{\theta} \ldots \longrightarrow \infty \ldots \tag{0.2}$$

where the symbol $\Re$ is the **MR** category representing the 'primordial' organism (PMR) and $\Re$[A,B] is the class of morphisms (proto-enzymes) bewteen the metabolic input class A (substrates) and the metabolic output class B (metabolic products of proto-enzymes). Note that in this linear sequence β represents a component capable of self-reproduction, such as a functional DNA double -helix molecule, that also acts as a template for shorter RNA molecules. On the other hand, the ribozyme θ is capable of both catalizing and 'reverse' encoding its RNA template into a more stable DNA duplex, ∞. One can reasonably expect that such primordial genes were at least partially conserved throughout evolution and may therefore be found through comparative, functional genomic studies. The first ribozymes may have evolved under high temperature conditions near cooling volcanoes in hot water springs and their auto-catalytic capabilities may have been crucial for rapidly producing a large population of self-reproducing primordials and their descendant, *Archea*-like organisms.

Note that the primordial defined here **MR**, or $PMR = \Re$, is an auto-catalytic, self-reproducing and autopoietic system; as shown by Warner in 1979, it can also be represented as a classical automaton [264] (see also [23] and [16]-[17]). At this stage, epigenetic controls have not yet been developed [32]. The PMR's 'evolution' is not yet entailed, or enabled; to entail further PMR development one also needs to provide it with a variable biogroupoid, a variable category, or an extended topos structure [27],[32], as further explained in the next sections.

5.6.4. *An Example of an Emerging Super-Complex System as A Quantum–Enzymatic Realization of the Simplest (**M,R**)–System.*

Note that in the case of either uni-molecular or multi-molecular, *reversible* reactions one obtains a *quantum-molecular groupoid*, QG, defined in terms of the variable molecular classes, or molecular class variables (mcv) and their *mcv–observables* [22],[24], a generalisation of the concept of molecular sets [22]-[24],[42]-[44]. The mcv concept extends and expands the scope of molecular set theories [42]-[44]. In the case of an enzyme, E, with an activated complex, $(ES)^*$, a *quantum biomolecuar groupoid* can be uniquely defined in terms of mcv–observables for the enzyme, its activated complex $(ES)^*$ and the substrate S. Quantum tunnelling in $(ES)^*$ then leads to the separation of the reaction product and the enzyme E which enters then a new reaction cycle with another substrate molecule S', indistinguishable–or equivalent to–S. By considering a sequence of two such reactions coupled together,

$$QG_1 \leftrightarrows QG_2$$

, corresponding to an enzyme f coupled to a ribozyme ϕ, one obtains a *quantum–molecular realization of the simplest* ***(M,R)****–system* (f, ϕ); see also the previous subsection for further details about the simplest primordial **(MR)**-system or **PMR**. The caveat here is that all relational systems considered above are *open* ones, exchanging both energy and *mass* with the system's environment in a manner which is dependent on time, for example in cycles, as the system 'divides'–reproducing itself; therefore, even though generalized quantum-molecular observables can be defined as specified above, neither a stationary nor a dynamic Schrödinger equation holds for such examples of 'super-complex' systems. Furthermore, instead of just energetic constraints–such as the standard quantum Hamiltonian–one has the constraints imposed by the diagram commutativity related to the mcv–observables, canonical functors and natural transformations, as well as to the concentration gradients, diffusion processes, chemical potentials/activities (molecular Gibbs free energies), enzyme kinetics, and so on. Both the canonical functors and the natural transformations defined above for uni- or multi-molecular reactions represent the relational increase in complexity of the emerging, super-complex dynamic system, such as, for example, the simplest (**M,R**)-system, (f, ϕ). Whereas f may represent a metabolic enzyme, the morphism, or map ϕ would represent in this model either an RNA-type molecule or a 'ribozyme', that is, a more complex molecule than f, which can then catalize the biosynthesis of f.

Further evolution of the **PMR** requires also the introduction of at least one *genetic duplication map* $\beta : H(A, B) \rightarrow H(B, H(A, B))$, representing more complex processes such as DNA duplication [235]-[236], and also a telomerase-based copying process for resetting the ends of the chromosomes, represented by a morphism $\theta : H(B, H(A, B)) \rightarrow H[H(A, B), H[B, H(A, B))]$ [22],[26]-[30], where A and B are, respectively, the input and output molecular sets of a metabolic enzyme f [19],[21]-[23],[235]-[236], as also shown above in diagram (0.3). Thus, a completely entailed, 'multicellular' **(M,R)**–system which can reproduce 'indefinitely' must have the extended, functional form: (f, ϕ, β, θ) [22],[26]-[32]. This theme of biological evolution will be now considered in more detail in the next section.

5.7. *Evolution and Dynamics of Systems, Organisms and Bionetworks: The Emergence of Increasing Complexity through Speciation and Molecular 'Evolution'/Transformations.*

Although Darwin's Natural Selection theory has provided for more than 150 years a coherent framework for mapping the Evolution of species [202], it could not attempt to explain how Life itself has emerged in the first place, predict the rates at which evolution occurred/occurs, or even predict to any degree of detail what the intermediate 'missing links', or intervening species, looked like, especially during their ascent to man. On the other hand, Huxley, the major proponent of Darwin's Natural Selection theory of Evolution, correctly proposed that the great,'anthropoid' apes were perhaps 10 million years ago in man's ancestral line.

We note here that part of the answer to the question how did life first emerge on earth is suggested by the modelling diagram considered in Section 3 and the evolutionary taxonomy: it must have been the simplest possible organism, i.e., one that defined the minimum conditions for the emergence of life on earth. Additional specifications of the path taken by the emergence of the first super-complex living organism on earth, the 'primordial', come from an extension of **MR** theory, and the consideration of its possible molecular realizations [16]-[17], and molecular evolution [156]-[157]. The question still remains open: why primordial life–forms or super-complex systems no longer emerge on earth, again and again? The usual 'answer' is that the conditions existing for the formation of the 'primordial' no longer exist on earth at this point in time. Even though Evolutionary theories aim to encompass all organisms and species, their focus is on eukaryotic, multi–cellular organisms. There are very substantial differences, however between both the cellular and genome structures of prokaryotes and eukaryotes. Furthermore, bacteria and Archea are the oldest and most numerous surviving organisms on earth despite of their much simpler structures. The variability of living systems is so great, however, that organisms could evolve above the microscopic scale of bacteria, Archea and most uni-cellular algae. Because of the very rapid division rate of microorganisms and the very high 'evolutionary pressures' they are exposed to, the evolution of new strains of microrganisms can be now observed both in nature and in the laboratory; man has become able to control or directly generate new strains of microorganisms through genetic engineering and artificial selection. In spite of such progress being made, this does not mean at all that our understanding of bacterial life is anywhere close to being complete. In fact, in the 'race for survival' between man and antibiotic–resistant bacteria, the latter seem to be gaining new ground.

5.8. *'Historical Continuity' in the Evolution of Super-Complex Systems: Topological Transformations and Discontinuities in Biological Development.*

Anthropologists and evolutionary biologists in general have emphasized biological evolution as a *'continuous'* process, in a *historical*, rather than a topological, or dynamic sense. This means that there are historical sequences of organisms–phylogeny lines– which evolved in a well-defined order from the simpler to the more complex ones, with intermediate stages becoming extinct in the process that translates 'becoming into being', as Prigogine might have said. This picture of evolution as a 'tree of life', due initially and primarily to Wallace and Darwin, subsequently supported by many evolutionists, is yet to be formulated in *dynamic*, rather than historical, terms. Darwin's theory of *gradual* evolution of more complex organisms from simpler ones has been subject to a great deal of controversy, which is still ongoing. If one were to accept for the moment Darwin's gradual evolution of species– instead of organisms– then, one may envisage the emergence of higher and higher *sub-levels* of super-complexity through biological evolution until a transition occurs through human society co-evolution to ultra-complexity, the emergence of human consciousness [91]. Thus, without the intervention of human society co-evolution, a smooth increase in the degree of super-complexity takes place only until a distinct and discrete transition to the (higher) ultra-complexity level becomes possible through society co-evolution. If the previous process of increasing complexity–which occurred before the transition at the super-complexity level– were to be iterated also at the ultra-complex level, one might ask how and what will be the deciding factor for the further 'co-evolution of minds' and the transition towards still higher complexity levels? Of course, one might also ask first the contingent ontology question if any such higher level above human consciousness could at all come into existence? As shown in one of our recent reports [33], the emergence of levels, or sub-levels, of increasing higher complexity can be represented by means of *variable* structures of increasingly higher order or dimensions. There remains also the unresolved question why humans –as well as parrots–have the *inherited* inclination to talk whereas the apes do not; thus, a chimpanzee pup will not talk even if brought up in a human environment, whereas a human baby will first 'babble' and than develop early a 'motherese' talk as an intermediate stage in learning the adults' language; the chimpanzee pup never babbles nor develops any 'motherese' through natural interactions with either its own biological mother or with a human, surrogate mother. These facts seem to point to the absence in apes of certain brain structures, perhaps linked to mirror neurons [200] , that are responsible for the human baby's inheritable *inclination* to babble (Wiener in ref. [269]), which then leads to speech through learning and nurture in the human environment. Unlike physical and chemical studies, evolutionary ones are usually limited severely by the absence of controlled experiments to yield the prerequisite data needed for a complete theory. The pace of discoveries is thus very much slower in evolutionary studies than it is in either physics or chemistry. Moreover, the timescale on which we know that biological evolution has occurred (and may still continue to occur), is extremely far from that of physical and chemical processes occurring on Earth, despite Faraday's saying that *"life is but a delayed chemical reaction"*. The 2-billion year timescale for biological evolution is a significant part of the evolution of the known universe itself over some 18 billion years. Thus, interestingly, both Evolutionary and Cosmological studies work by quite different ontological and epistemologic means to uncover events that span across enormous spacetime regions by comparison with either a human's lifespan or the entire history and pre-history of humanity. Whereas in Cosmology the view of an *absolute and fixed* Universe prevailed for quite a long time, it is currently accepted that the Universe 'evolves' as well as keeps rapidly inflating– it changes while very rapidly expanding relative to the observer or reference frame. Astrophysical studies have now established that our observable Universe is neither fixed nor absolute (thus validating Spencer's contention in 1862 of the absence of absolute space and time). On the one hand, Cosmology benefits from the use of very powerful physical means to investigate the Universe both experimentally and theoretically. On the other hand, evolutionary biology is limited mostly to indirect means of deduction, and also much fewer means of actual experimentation. This is undoubtedly one major reason, but not the only one, why Darwin's over-simplifying concepts of Natural Selection and Origin of species have survived for a surprisingly long time in biology, and are still considered by many biologists as well-established 'fact' even today. 'Survival of the fittest' seems to have been, however Herbert Spencer's contribution to 'explaining' biological evolution, as well as society's 'evolution' (in Spencer's published opinion). On a much smaller space and time scale than Cosmology, biological evolution has generated a vast number of species, however, with the majority of the species becoming extinct; the species

survival rate is estimated to be below 1%. In this latter process, geographical location, the climate, as well as occasional catastrophes (glaciary eras, fires, meteorites, volcanoes, and surely disease, etc.), seem to have also played significant roles in reducing the species survival rates, in addition to competition for survival within the same niche. The historical view of biological evolution proposed by Darwin stems from the fact that every organism, or living cell, usually originates only from just a single cell, or egg, and there is no *de nuovo* re-starting of biological evolution. This raises two very important, related questions:
1. What where the initial conditions required for life to start on Earth in the first place?
and
2. How did the first, primordial organism emerge a few billion years ago, and in what structural-functional form?

We shall see briefly how specific organismic models may provide some partial answers to these key questions that were left completely unanswered until now by Darwin's theory, or indeed any of its reductionist alternatives by neo-Darwinists who assume only a gradual evolution of species.

5.9. *Biological Species. Evolving Species as Variable Biogroupoids*

After a century-long debate about what constitutes a biological species, taxonomists and general biologists seem to have now adopted the operational concept proposed by Mayr in 1970 [184]:

> *"a species is a group of animals that share a common gene pool and that are reproductively isolated from other groups."*

Obviously, this definition is not to be interpreted as a genomic identity of all the organisms within any given species, as there are relatively small genetic differences between individuals of the same species, in addition to those related to gender that are significant (such as, XX vs. XY chromosomes). Unfortunately, this definition is not readily applicable to extinct species and their fossils, the subject of great interest to paleoanthropologists, for example. From an ontology viewpoint, the biological species could be defined as *a class of equivalent organisms with regard to sexual reproduction and/or all genes of the functional genome that determine the key physicological functions*, or algebraically as a *biogroupoid** [32]. Then, one has the algebraic representation of a species as *a biogroupoid of organisms that share a common genome and that are reproductively isolated from other organisms.* Undoubtedly, further refinements of this definition are also possible; for example, one would have to represent also algebraically the condition that the organisms of the same species/biogroupoid are 'reproductively isolated' from the organisms of another species represented by a different biogoupoid. Future refinements may also include a mathematical representation of the epigenetic memory that is needed to preserve the somatic progenitor state through repeated cell divisions; predictions from models based on such network

*We understand *biogroupoids* to be defined as the groupoids of equivalence relations in biological systems, for example, of biomolecular systems/networks/graphs that with concatentation of paths can be reduced to equivalence class types based on arrow/vertex types, such as 'neurogroupoids' for neural networks and circuitry [23],[32],[264].

representations might be useful in resetting the pathological epigenetic memory involved in certain cancers [27],[30]-[32],[35].

As satisfactory as taxonomic tools these two definitions might be, they are not directly useful for understanding how evolution occurs. The biogroupoid concept, however, has the advantage that it can be readily extended, or generalised, to more flexible mathematical concepts, such as that of a *variable biogroupoid*, which can be then utilized in theoretical evolutionary studies. Thus, through theoretical predictions, one could impact on empirical evolutionary studies or on artificial selection experimentation, as well as possibly on organismal taxonomy and ontology. Other uses may be in anthropological studies of a series of species by homotopic or homological transformations that are much more general than the analytical coordinate transformations introduced for this purpose, and also tested, by D'Arcy Thompson in collaboration with his specialised Dutch coworker who carried out the coordinate transformations for comparing the skulls of animals from different species [259].

5.9.1. *Variable Biogroupoids and Fibrations*

For a collection of *variable groupoids* we can firstly envisage a defintion in terms of a parametrized family of groupoids $\{\mathsf{G}_\lambda\}$ with parameter λ (which may be a time parameter, although in general we do not insist on this). This is one basic and obvious way of seeing a variable groupoid structure. If λ belongs to a set M, then we may consider simply a projection $\mathsf{G} \times M \longrightarrow M$, which is an example of a trivial fibration. More generally, we could consider a *fibration of groupoids* $\mathsf{G} \hookrightarrow Z \longrightarrow M$ [140].

However, we expect in several of the situations discussed in this paper (such as, for example, the metabolic groupoid introduced in the previous subsection) that the systems represented by the groupoid are interacting. Thus, besides dynamic or general systems modelled in terms of a *fibration of groupoids* [140], we may alternatively consider a *multiple groupoid* defined as a set with a number of groupoid structures any distinct pair of which satisfy an *interchange law*; the latter can be expressed as follows: each pair is a morphism for the other, or alternatively, there is a unique expression of the following composition:

$$\begin{bmatrix} x & y \\ z & w \end{bmatrix} \quad \begin{array}{l} \longrightarrow j \\ \downarrow \\ i \end{array} \tag{0.3}$$

where i and j must be distinct for this concept to be well defined. This uniqueness can also be represented by the equation

$$(x \circ_j y) \circ_i (z \circ_j w) = (x \circ_i z) \circ_j (y \circ_i w). \tag{0.4}$$

This example presented by Ronald Brown in [69] illustrates the principle that a 2-dimensional formula may be more comprehensible than a linear one! Thus, Brown and Higgins showed in 1981 that certain multiple groupoids equipped with an extra structure called *connections* were equivalent to another structure called a *crossed complex* (cited in [64]),[68]), which had already occurred in homotopy theory as *crossed module and crossed complex* [63]-[64],[71]-[72],[75], where it plays an important role.

For example, the notion of an *atlas* of structures should, in principle, apply to many interesting, topological and/or algebraic, structures: groupoids, multiple groupoids, but basically those with some form of 'objects' which give the geography for the patching of overlaps in an atlas. Another example provided by R. Brown in 2007 [69], which may also involve multiple groupoids– in the ultra-complex system of the human mind– is that of *synaesthesia*–the case of extreme communication processes between different types of 'logics' or different levels of 'thoughts', or thought processes. The key point here is that of *interactive communication*. Hearing has to communicate to sight/vision in some way; this seems to happen in the human brain in the audiovisual (neocortex) and in the Wernicke (W) integrating area in the left-side hemisphere of the brain, that also communicates with the speech centers or the Broca area, also in the left hemisphere.

The very common health problem caused by the senescence of the brain could be approached as a *local-to-global*, super-complex ageing process represented. It would be very interesting to find real ways in which higher categories and groupoids could help the analysis of complex biological networks. Aging, as surmised by Rosen in 1987 [232], seems to be not a local but a *global*, senescence, super-complex dynamic process, and this is consistent with a COLP-type process involving multiple failures rather than a single specific cause or mechanism.

It is interesting that Connes states in [85] that Heisenberg discovered quantum mechanics by considering the convolution algebra associated to the groupoid of transitions of the hydrogen spectrum: Born pointed out that this algebra was the well-known matrix algebra. It thus seems likely that more subtle applications in physics and systems theory algebraic structures will be found by considering double groupoids first of transitions, and then of more complicated situations, such as those associated with crossed modules.

Developmental processes, and in general, ontogeny–considered from a structural or anatomical viewpoint– involves not only geometrical or topology–preserving transformations but more general/complex transformations of much more flexible structures such as the variable groupoids. The natural generalisations of variable groupoids lead to 'variable topology' and variable category concepts that are considered in the next subsections.

5.10. *Super–Complex Network Biodynamics in Variable Biogroupoid Categories. Variable Bionetworks with Variable Topology and their Super-Categories*

This subsection is an extension of the previous one in which we introduced variable biogroupoids in relation to speciation and the evolution of species. The variable category concept generalizes that of variable groupoid which can be thought as a variable category whose morphisms are invertible; the latter is thus a more 'symmetric' structure than the general variable category. Variable biogroupoids are also good models of biosystems–super-complex systems that in general have a varying topological structure, or variable topology. Thus, we realize here the basic reason for which organisms are super–complex: their dynamics can only be adequately characterized through a variable topology, or 'super–topology', HDA, etc., generated by emergent meta-processes of processes. We have already seen that variable biogroupoid and COLP representations of biological species can provide powerful tools for tracking evolution at the level of species. On the other hand, the representation of multi-cellular organisms is likely to require more general structures, and super-structures of structures– in a relational rather than an anatomical, or biological, microstructure sense [11]-[13].

In other words, this leads towards higher-dimensional algebras (HDAs) representing the super-complex hierarchies present in a complex–functional, multi-cellular organism, or in a highly-evolved functional organ such as the human brain. The latter (HDA) approach will also be discussed in the last section in relation to neurosciences and consciousness, whereas we shall address next the question of representing organisms regarded as (dynamic) biosystems in terms of variable categories that are lower in complexity than the ultra-complex human mind. The range of applications for variable categories includes neurosciences, neurodynamics and brain development [32], in addition to the evolution of the simpler genomes and/or interactomes [35]. Ultimately, it does lead directly to the more powerful 'hierarchical' structures of higher dimensional algebra.

5.11. *Variable Topology Representations of Bionetwork Dynamic Complexity*

Let us recall the basic principle that a *topological space* consists of a set X and a 'topology' on X [58], where the latter gives a precise but general sense to the intuitive ideas of 'nearness' and 'continuity'. Thus, the initial task is to axiomatize the notion of 'neighborhood' and then consider a topology in terms of open or of closed sets, a compact-open topology, and so on [58], [63]. In any case, a topological space consists of a pair $(X, \mathcal{T})$ where $\mathcal{T}$ is a topology on X. For instance, suppose an *open set topology* is given by the set $\mathcal{U}$ of prescribed open sets of X satisfying the usual axioms (Chapter 2 in [63]). Now, to speak of a variable open-set topology one might conveniently take in this case a family of sets $\mathcal{U}_\lambda$ of *a system of prescribed open sets*, where λ belongs to some indexing set Λ. The system of open sets may of course be based on a system of contained neighbourhoods of points where one system may have a different geometric property compared say to another system (a system of disc-like neighbourhoods compared with those of cylindrical-type).

D8. *Topological space with variable topology.*

In general, we may speak of a *topological space with a varying topology* as a pair $(X, \mathcal{T}_\lambda)$ where $\lambda \in \Lambda$. The idea of a varying topology has been introduced to describe possible topological distinctions in bio-molecular organisms through stages of development, evolution, neo-plasticity, etc. This is indicated schematically in the diagram below where we have an n-stage dynamic evolution (through complexity) of categories D_i where the vertical arrows denote the assignment of topologies $\mathcal{T}_i$ to the class of objects of the D_i along with functors $\mathcal{F}_i : \mathsf{D}_i \longrightarrow \mathsf{D}_{i+1}$, for $1 \leqslant i \leqslant n-1$:

$$\begin{array}{ccccccc} \mathcal{T}_1 & & \mathcal{T}_2 & \cdots & \mathcal{T}_{n-1} & & \mathcal{T}_n \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ \mathsf{D}_1 & \xrightarrow{\mathcal{F}_1} & \mathsf{D}_2 & \xrightarrow{\mathcal{F}_2} & \cdots\ \mathsf{D}_{n-1} & \xrightarrow{\mathcal{F}_{n-1}} & \mathsf{D}_n \end{array}$$

In this way, a variable topology can be realized through such n-levels of complexity of the development of an organism. Another instance is when cell/network topologies are prescribed and in particular when one considers a categorical approach involving concepts such as *the free groupoid over a graph* [65]. Thus a *varying graph* system clearly induces an accompanying system of variable groupoids. As suggested by Golubitsky and Stewart in 2006, symmetry groupoids of various cell networks would also appear relevant to the physiology of animal locomotion [124]. However, such examples are not limited to locomotion, and examples of symmetry groupoids abound in various cellular systems.

5.12. *Quantum Genetic Networks and Microscopic Entropy*

Following Schrödinger's early attempt in 1945 [229], Robert Rosen's report in 1960 was perhaps one of the earliest quantum–theoretical approaches to genetic problems that utilized explicitly the properties of von Neumann algebras and spectral measures/self–adjoint operators [231]. A subsequent approach considered genetic networks as *quantum automata* and genetic reduplication processes as *quantum relational oscillations* of such bionetworks [13]. This approach was also utilized in subsequent reports to introduce representations of genetic changes that occur during differentiation, biological development, or oncogenesis in terms of *natural transformations of organismal (or organismic) structures* [19],[21],[23],[28], thus paving the way to a *Quantum Relational Biology* [26],[31],[35]. The significance of these results for quantum bionetworks was also recently considered from both a *logical and an axiomatic* viewpoint [36]. On the other hand, the extension of quantum theories, and especially quantum statistics, to non-conservative systems, for example by Prigogine has opened the possibility of treating *irreversible*, super-complex systems that vary in time and 'escape' the constraints of unitary transformations, as discussed above. Furthermore, the latter approach allows the consideration of functional genetic networks from the standpoint of quantum statistical mechanics and microscopic entropy. Thus, information transfer of the 'genetic messages' throughout repeated somatic cell divisions may be considered either in a modified form of Shannon's theory of communication channels in the presence of 'noise', or perhaps more appropriately in terms of Kolmogorov's concept of entropy [170].

On the other hand, the preservation and/or repeated 'transmission' of genetic 'information' through germ cells– in spite of repeated quantum 'observations' of active DNA genes by replicase– is therefore an open subject that might be better understood by employing the concept of microscopic entropy in Quantum Genetics.

5.13. *Lukasiewicz and LM-Logic Algebra of Genome Network Biodynamics. Quantum Genetics, Q-Logics and the Organismic LM–Topos*

The representation of categories of genetic network biodynamics **GNET**s as subcategories of LM–Logic Algebras (**LMA**s) was recently reported in [36] and several theorems were discussed in the context of morphogenetic development of organisms. The **GNET** section of the cited report was a review and extension of an earlier article on the 'immanent' logic of genetic networks and their complex dynamics and non-linear properties [24]. Comparison of GNET universal properties relevant to *Genetic Ontology* can be thus carried out by colimit- and/or limit– preserving functors of GNETs that belong to adjoint functor pairs [15],[23],[30]-[32],[154],[213],[256].

Furthermore, evolutionary changes present in functional genomes can be represented by natural transformations of such universal–property preserving functors, thus pointing towards evolutionary patterns that are of importance to the emergence of increasing complexity through evolution; they can also lead to the emergence of the human organism. Missing from this approach, however, is a consideration of the important effects of social, human interactions in the formation of language, symbolism, rational thinking, cultural patterns, creativity, and so on... to full human consciousness– as we know it.

5.13.1 *The Organismic LM–Topos*

As reported previously by Baianu et al. in [32]) it is possible to represent directly the actions of LM, many-valued logics of genetic network biodynamics in a categorical structure generated by selected LM–logics. The combined logico-mathematical structure thus obtained may have several operational and consistency advantages over the GNET-categorical approach of 'sets with structure'. Such a structure was called an 'LM-Topos' and represents a significant, non-commutative logic extension of the standard Topos theory which is founded upon a commutative, intuitionist (Heyting-Brouwer) logic. The non-commutative logic LM–topos offers a more appropriate foundation for structures, relations and organismic or societal functions that are respectively super-complex or ultra-complex. This new concept of an LM–topos thus paves the way towards a Non-Abelian Ontology of highly complex spacetime structures as in organisms and societies.

5.14. *Natural Transformations of Evolving Organismic Structures*

5.14.1 *Generalized (M,R)-Systems as Variable Groupoids.*

We have considered the important example of MR-Systems with *metabolic groupoid* structures (that is, *reversible enzyme reactions/metabolic functions–repair replication* groupoid structures), for the purpose of studying RNA, DNA, epigenetic and genomic functions. For instance, the relationship of

$$\text{METABOLISM} = \text{ANABOLISM} \Longrightarrow \;\; \Longleftarrow \text{CATABOLISM}$$

can be represented by a metabolic groupoid of *'reversible', anabolic/catabolic processes.* In this respect, the simplest MR-system can be represented as a *topological groupoid* with the open neighbourhood topology defined for the entire dynamical state space of the MR-system, that is an open/generic– and thus, a structurally stable– system, as defined by Rosen's dynamic realizations of MR-systems [235],[236]. This requires a descriptive formalism in terms of *variable groupoids* following which the human MR-system would then arise as the *colimit* of its complete biological family tree expressible in terms of a family of many linked/connected groupoids; this variable biogroupoid representation proves also to be useful in studies of evolution.

5.14.2 *A Simple Metabolic-Repair **(M,R)**–System with Reverse Transcription: An example of Multi-molecular Reactions Represented by Natural Transformations.*

We shall consider again the diagram corresponding to the simplest (**M, R**)-System realization of a Primordial Organism, PO. The RNA and/or DNA duplication and cell divisions would occur by extension to the right of the simplest MR-system, (f, Φ), through the $\beta : H(A,B) \to H(B,H(A,B))$ and $\theta\colon H(B,H(A,B)) \to H(H(A,B),H(B,H(A,B)))$ morphisms. Note in this case, the 'closure' entailed by the functional mapping, θ, that physically represents the regeneration of the cell's *telomere* thus closing the DNA-loop at the end of the chromosome in eukaryotes. Thus θ represents the activity of a *reverse transcriptase.* Adding to this diagram an hTERT suppressor gene would provide a *feedback* mechanism for an effective control of the cell division and the possibility of cell cycle arrest in higher, multi-cellular organisms (which is present only in *somatic* cells). The other alternative-which is preferred here-is the addition of an hTERT *promoter gene* that may require to be activated in order to begin cell cycling [35]. This also allows one to introduce simple models of carcinogenesis or cancer cells. Rashevsky's hierarchical theory of organismic sets can also be constructed by employing mcv's with their observables and natural transformations as it was shown by Baianu in 1980 [19].

*Thus, one obtains by means of natural transformations and the Yoneda-Grothendieck construction a unified, categorical-relational theory of organismic structures that encompasses those of organismic sets, biomolecular sets, as well as the general (**M,R**)-systems/autopoietic systems which takes explicitly into account both the molecular and quantum levels in terms of molecular class variables* [22]-[28],[41]-[44].

5.15. *Oncogenesis, Dynamic Programming and Algebraic Geometry Models of Cellular Controls*

In this section we shall discuss changes of normal controls in cells of an organism. It was previously proposed that certain specific changes of cellular controls occur in oncogenesis as a result of an initial abnormal human genome architecture [13],[21],[23],[26]-[28], [30],[32],[35].

These changes may become permanent, if the basic relational oscillators of the cell have also been modified. In the language of qualitative dynamics this may be translated as a change of dominating attractors, followed by the inhibition or destruction of the former dominating attractors. This kind of change is not necessarily a mutation, that is, the change may not produce the replacement of some essential observables in the genetic system; this would however result eventually in many mutations and also alter the chromosomal architecture and modify the diploid arrangement of chromosomes in the cell nucleus. This may be the reason for which extensive research on cancers failed to discover so far a general, unique and specific alteration of the genetic system of cancer cells, *except for aneuploidy.* The change of basic relational oscillators in the genetic system may have such consequences as, for example, abnormally large nucleoli or major chromosomal aberrations. The reason may be that a change in the subspace of the cellular dynamic controller–such as p54– produces the change of dynamic programming of the whole cell. Dynamic programming consists in the existence of distinguished states, or policies in the subspace corresponding to the controller, to which correspond specific changes of trajectories in the subspace of the controlled subsystem.

5.16. *Evolution as a Local-to- Global Problem: The Metaphor of Chains of Local Procedures. Bifurcations, Phylogeny and the 'Tree of Life'.*

Darwin's theory of natural selection which considers both specific and general biological functions such as adaptation, reproduction, heredity and survival, has been substantially modified and enriched over the last century. In part, this is due to more precise mathematical approaches to population genetics and molecular evolution which developed new solutions to the key problem of speciation [50],[131],[184]-[185],[212],[245], but also some major conceptual advances as well [156]-[157].

Modified evolutionary theories include neo–Darwinism, the 'punctuated evolution', notably by Gould in 1977 [128]-[129] and the 'neutral theory of molecular evolution' of Kimura reported in 1983 [156]-[157]. The latter is particularly interesting as it reveals that evolutionary changes do occur much more frequently in unexpressed, or silent, regions of the genome, thus being 'invisible' phenotypically. Therefore, such frequent changes (or 'silent mutations') are uncorrelated with, or presumed to be unaffected by, natural selection. On the other hand, rather infrequent *polygenic* mutations do occur bringing about relatively rapid, *multiple change*s on the evolutionary timescale on the order of 500 years instead of hundreds of thousands or millions of years. Such polygenic mutations that have distinct survival advantages for individuals of a species spread rapidly throughout the population leading to the formation and stability of the new species thus formed.

For further progress in completing a logically valid and also experimentally-based evolutionary theory, an improved understanding of both speciation and species epigenetic stability is required, together with substantially more extensive, experimental/genomic and epigenetic data related to speciation than it is currently available. Furthermore, the ascent of man, is apparently not the result of only natural selection, but also that of co-evolution through societal interactions [91]. Thus, simply put: the emergence of human speech and consciousness occurred both through selection and co-evolution [91], with the former not being all that 'natural' because society played a protective, as well as selective, role from the very beginnings of hominin and hominid societies for longer than 2.2 million years. Somewhat surprisingly, the subject of *social selection* in human societies is rarely studied even though it may have played a crucial role in the emergence of *H. sapiens*, and occurs in every society that we know, without any exception.

Furthermore, there is a theory of levels, ontological question that has not yet been adequately addressed, although it has been identified: *at what level does evolution operate: species, organism or molecular (genetic)?* According to Darwin the answer seems to be the species. However, not everybody agrees with his idea because in Darwin's time a valid theory of inherited characters–or genetics– was neither widely known nor accepted. Moreover, molecular evolution and concerted mutations are quite recent concepts whose full impact has not yet been realized. As Brian Goodwin [126] put it succinctly in 1982:

"Where has the organism disappeared in Darwin's evolutionary theory?"

The answer in both Goodwin's opinion, and also in ours, lies in the presence of key functional/relational patterns that emerged and were preserved in organisms throughout various stages over billions of years of biological evolution. The fundamental relations between organism, species and the speciation process itself do need to be directly considered by any theory that claims to explain the evolution of species and organisms. Furthermore, an adequate consideration of the biomolecular levels and sub-levels involvement in speciation and evolution must also be present in any improved evolutionary theory. These fundamental questions were recently considered from this categorical ontology viewpoint in [33]-[34].

In his widely read book, D-Arcy W. Thompson [259] gave a large number of biological examples of organismic growth and forms analyzed at first in terms of physical forces. Then, he was successful in carrying out analytical geometry coordinate transformations that allow the continuous, homotopic mapping of series of species that are thought to belong to the same branch–phylogenetic line– of the tree of life. However, he found it very difficult or almost impossible, to carry out such transformations for fossils with skeleton remains of species that belonged to different evolutionary branches. Thus, he arrived at the conclusion that the overall evolutionary process is *not a continuous sequence* of organismic forms or phenotypes (see p. 1094 of his book [259]), which indeed it may not be the case.

Thus, one needs to address the question of super-complex systems' evolution as a *local-to-global* problem instead of a topologically continuous process. We are then seeking solutions in terms of the novel categorical concepts that were sketched in the previous subsections and also more precisely defined in [33] and [69]. Therefore, we consider here biological evolution by introducing the unifying metaphor of *'local procedures'* which may represent the formation of new species that branch out to generate still more evolving species. Because genetic mutations that lead to new species are discrete changes, we are therefore not considering evolution as a series of continuous changes–such as a continuous curve drawn analytically through points representing species –but heuristically as a *tree of 'chains of local procedures'* [71]. Evolution may be alternatively thought of and analyzed as a *composition of local procedures.* Composition is a kind of combination, and so it might be confused with a colimit, but they are substantially different concepts. Therefore, one may attempt to represent biological evolution as an evolutionary tree, or 'tree of life', with its branches completed through chains of local procedures (pictured in Figure 1 as overlapping circles) involving certain groupoids, previously defined as *variable topological biogroupoids* in [33],[39]. The overlaps in this latter representation correspond to 'intermediate' species or classes/populations of organisms which are rapidly evolving under strong evolutionary pressure from their environment (including competing species, predators, etc., in their niche).

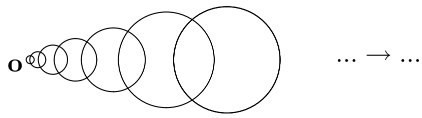

Figure 1. A pictorial representation of Biological Evolution as a composition of local procedures involving variable biogroupoids that represent biological speciation phenomena. COLPs may form the branches of the evolutionary tree, oriented in this diagram with the time arrow pointing to the right. The overlaps would however be far greater than this figure would indicate as a mere geometrical metaphor.

The notion of 'local procedure' is an interpretation of Ehresmann's formal definition of a *local admissible section* s for a groupoid G in which $X = \mathrm{Ob}(G)$ is a topological space. Then s is a section of the source map $\alpha : G \to X$ such that the domain of s is open in X. If s, t are two such sections, their composition *st* is defined by $\mathsf{st}(x) = \mathsf{s}(\beta t(x)) \circ t(x)$ where $\circ$ is the composition in G. The domain of **st** could also be empty. One may also put the additional condition that s is 'admissible' , namely $\beta\mathsf{s}$ maps the open domain of s homeomorphically to the image of $\beta\mathsf{s}$, which itself is open in X. Then an admissible local section is *invertible* with respect to the above composition. A tree-graph that contains only single-species biogroupoids at the 'core' of each 'local procedure' does define precisely an evolutionary branch without the need for subdivision because a species is an 'indivisible' entity from a breeding or reproductive viewpoint. Several different concepts in organismic dynamics, stability and variability 'converge' here on the metaphor of chains of 'local procedures' for evolving organisms and species. Such distinct representations are: the dynamic genericity of organismic states which lead to structural stability, the logical class heterogeneity of living organisms, and the inherent 'bio–fuzziness' of organisms in both their structure and function that was pointed out in 1968 [11]; alternatively, they can also considered as Maturana's autopoietic models of the 'structural variability' exhibited by living systems reported in 1980 [182], that are imposed to the organisms through their couplings with a specific environmental niche.

This novel, *dynamic* rather than historic/Darwinist intuition of evolution may be difficult to grasp at first as it involves several construction stages on different ontological levels: it begins with organisms (or possibly even with biomolecular categories), emerges to the level of populations/subspecies/ species that evolved into classes of species, that had then further evolved, ... and so on. Finally, it reaches the point in time where the emergence of man's, *Homo* family of species began to separate from other hominin/hominide families of species some 2.2 million years ago. One concludes, in agreement with Robert Rosen's ideas (personal communication to ICB in 1970), that the evolutionary processes operate on several different levels or sublevels of reality, on quite different time scales; it is now generally accepted that speciation is also aided by geographical barriers or geological accidents. This highly complex, dynamic reality of the emerging higher levels of complexity is quite different from that in Darwin's widely acclaimed "Origin of Species", and it is also a much more powerful concept than Spencer's vague evolutionary speculations [249] published in 1898; furthermore, it also includes– but is not limited to– Goodwin's excursions into contingent, 'chaotic complexity' [125]-[126]. The following subsection links up our novel evolutionary model with recently emerging autopoiesis models, and their earlier, corresponding Rosen's **MR-**systems.

5.17. *Autopoiesis Models of Survival and Extinction of Species through Space and Time*

The autopoietic model of Maturana and Varela [182] claims to explain the persistence of living systems in time as the consequence of their structural coupling or *adaptation* as structure–determined systems, and also because of their existence as *molecular* autopoietic systems with a 'closed' network structure. As part of the autopoietic explanation is the 'structural drift', presumably facilitating evolutionary changes and speciation. One notes that autopoietic systems may be therefore considered as dynamic realizations of Rosen's simple **MR** s. Similar arguments seem to be echoed more recently in 2003 by Dawkins [90] who claims to explain the remarkable persistence of biological organisms over geological timescales as the result of their intrinsic, (super-) complex, adaptive capabilities. The point is being often made that it is not the component atoms that are preserved in organisms (and indeed in 'living fosils' for geological periods of time), but the *structure-function relational pattern*, or indeed the associated organismic categories and supercategories [11]-[15]. This is a very important point: only the functional organismic structure or pattern persists as it is being conserved and transmitted from one generation to the next. Biomolecules turn-over in an organism, and not infrequently, but the *structure-function patterns/organismic categories remain unchanged*/are conserved over long periods of time through repeated repairs and replacements of the molecular parts that need repairing, as long as the organism lives. Such stable patterns of relations are, at least in principle, amenable to logical and mathematical representation without tearing apart the living system. Hence the relevance here, and indeed the great importance of the science of abstract structures and relations, i.e., Mathematics. In fact, looking at this remarkable persistence of certain gene subnetworks in time and space from the categorical ontology and Darwinian viewpoints, the *existence of live 'fossils'* (e.g., a coelacanth found alive in 1923 to have remained unchanged at great depths in the ocean as a species for 300 million years!) it is not so difficult to explain; one can attribute the rare examples of 'live fossils' to the lack of 'selection pressure in a very stable niche'. Thus, one sees in such exceptions the lack of any adaptation apart from those which have already occurred before some 300 million years ago. This is by no means the only long lived species: several species of marine, giant unicellular green algae with complex morphology from a family called the *Dasycladales* may have persisted as long as 600 million years [126]. However, the situation of many other species that emerged through *super-complex adaptations*–such as the species of *Homo sapiens*–is quite the opposite, in the sense of marked, super-complex adaptive changes over much shorter time scales than that of the exceptionally 'lucky' coelacanths. Clearly, some species, that were less adaptable, or just unlucky, such as the Neanderthals or *Homo erectus*, became extinct; neither of the two seem to have been capable of structured speech, as discussed next. The latter two distinct species of hominins seem to have co-existed at some of the locations with the *Homo sapiens* species for relatively short intervals of time, on the order of several tens of thousands of years, or even less. The consensus in the specialised literature is that these three distinct species have not, however, intermingled, or exchanged genes.

6. Human Consciousness and Society: Ultra-Complexity and Consciousness. The Emergence of *Homo sapiens*

We are briefly considering here the rather tenuous evidence for the emergence of the *Homo sapiens* species– the Ascent of Man. The related question of the development of syntactically–structured speech through social *co-evolution* [91] is also addressed in this section. Thus, the formation of the first human societies was apparently closely correlated with efficient communication through structured speech [186]; on the other hand, the propagation, further development and indeed elaboration of speech was both made possible and sustained only through social interactions in the pre-historic human societies [91],[186].

6.1. *Biological Evolution of Hominins (Hominides)*

Studies of the difficult problem of the emergence of man have made considerable progress over the last 50 years with a series of several key hominide/hominin fossils being found, such as: *Australopithecines, Homo erectus,* and *Homo habilis* being found, preserved, studied and analyzed in substantial detail. *Hominini* is defined as the tribe of *Homininae* that only includes humans (*Homo*), chimpanzees (*Pan*), and their extinct ancestors. Members of this tribe are called *hominins* (cf. hominidae or 'hominids'). Humans, on the other hand are: of the Kingdom: Animal; Phylum: Chordate; Class: Mammal; Order: Primate;...; Tribe: hominin. The Tribe of hominini describes all the human/human-like species that have ever evolved (including the extinct ones) which excludes the chimpanzees and gorillas. On the other hand, the corresponding, old terminology until the 1980s was 'hominides', now hominoides. Among these, *Homo erectus*

(and *H. ergaster*) were probably the first hominins to form a hunter gatherer society. Even though *H. erectus* used more sophisticated tools than the previous hominin species, the discovery of the Turkana boy in 1984 has produced the very surprising evidence that despite the *H. erectus*'s human-like skull and general anatomy, it was disappointingly incapable of producing sounds of the complexity required for either, ancient ($\prec$ 8,000 BC) or modern, elaborate speech. Thus, it seems that *H. erectus* may not have been anatomically capable of speech because it was still lacking the necessary vocal chords and mouth anatomical features required for speech. They probably nonetheless must have had communication through for example expression, gesture, and sound, in order to manage cooking and general survival. It has been reported † that the use of cooked food, and so of fire, was necessary for the particular physiognomy of even *H. erectus*, as against other primates, and such use perhaps required a societal context many millenia even before this hominin, partly in terms of the construction of hearths, which were a necessity for the efficient cooking of food. Thus we respond to the myth of Prometheus! There is however the caveat that artefacts such as tools have also been claimed by some paleoanthropologists to have belonged actually to *H. sapiens*, not to *H. erectus* or other hominins.

Thus, *H. sapiens* stands up as the only remaining species which is indeed *unique* in its mental/reasoning abilities, vocal apparatus and syntactically-structured, flexible speech. Other factors, such as the better use of purposefully designed tools, simple weapons and the intense struggle for the survival of the fittest have also contributed greatly to the selective advantages of *H. sapiens* in the fierce struggle for its existence; nevertheless, there is an overwhelming consensus in the specialised literature that the *co-evolution* of the human mind and society was the predominant, or key factor for the survival of *H. sapiens* over that of all other closely related species in the genus *Homo* that did not survive– in spite of having existed earlier, and some probably much longer than *H. sapiens*.

6.2. *The Ascent of Man through Social Co-Evolution. The Evolution of the Human Brain. Emergence of Human Elaborate Speech and Consciousness*

As stated above, there seems to be little doubt that a 'human–like' brain already may have been shaping up in *Homo erectus*, *ergastus*, or the Neanderthals‡, but none of these hominides and hominins are commonly thought to have been able to speak and generally communicate to the extent of forming a 'society of hominins' that could compete with the emerging *H. sapiens* 'primitive' societies. Therefore, such pre-human populations became extinct, presumably when the food supply could no longer support both hominin and increasing human populations in the same ecological niche. Following *Homo erectus*, there was some apparent but temporary slowing down of hominin biological evolution that may have occurred over the next 2 million years, or longer, for hominides other than *H. sapiens*; according to several anthropologists *H. sapiens* separated as a species from a common ancestor with *H. ergastus* about 2.2 million years ago. There seems to be no statement available in the literature about the latter's ability for structured speech, and thus it remains an open question.

Therefore, our assumption made here– based on the existing literature consensus– is that the human brain considered as a biological organ, or subsystem, has evolved *before* self-awareness and the highly coherent conscious states of the ordered mind of low informational 'entropy' level that emerged only later through social co-evolution [91],[132],[186].

†See Richard Wrangham, 'Catching fire: how cooking made us human', Profile Books, 2009. R. Bednarik has excavated a major hearth in S. Africa dating from c. 1.8 million years BP, well before *H. erectus*

‡Note, however, that the question of speech capacity of the Neanderthal hominin is still a subject of considerable debate. A paper by Louis-Jean Bora, Jean-Louis Heimb, Kiyoshi Hondac and Shinji Maedad, in Journal of Phonetics Volume 30, Issue 3, July 2002, 465-484, disagrees strongly with other work in this area.

The human mind is therefore proposed here to be represented by an *ultra–complex* 'system of processes' based on, *but not necessarily reducible to*, the human brain's super-complex level of activities that both enable and entail the emergence of the human mind's own consciousness. Thus, an attempt is made here to both define and represent in categorical ontology terms the human consciousness as an *emergent/global, ultra-complex process* of mental activities as distinct from–but correlated with–a multitude of integrated local super-complex processes that occur in the human brain. It has been suggested –with some evidence from neurophysiological experiments– that mirror neurons may mediate the social interactions leading to coherent, rational and elaborate speech, that thereafter supports the emergence of consciousness. Thus, the emergence of symbolic language with syntax, and the whole social co-evolution and progression towards consciousness may have accelerated only through the *unique* appearance of *H. sapiens* [91],[186]. The faculty of speech may, or could, have however predated the phylogenetic separation of the human population. It is generally accepted that syntactically-structured language is essential to the communication between humans in the society at large, and it is also central to the sense of national identity and identification of cultures and ethnic groups. Data networks are therefore very important to the continued development of language. Linguistics studies and analyses the structure of human language and the relationships among such languages. It is interesting in this respect that Sign Language, which was developed for, and by, deaf children, is also regarded as an established language, with its special structure.

Other hominin species, such as for example the Neanderthal species, may not have been able to compete with *H. sapiens* because they did not evolve beyond very primitive, small hunter–gatherer groups, although it is generally recognised that the Neanderthals became smart tool makers and users. Stronger evidence for the appearance of the coherent human speech comes only from the discoveries of the pre-historic Cro–Magnon man that lived some 60,000 years ago. Most anthropologists agree that the Cro-Magnon belongs to the *Homo sapiens* species. This leads one to conclude that a relatively rapid 'transition' either occurred or began *from super- to ultra- complexity*, from biologically-based evolution to the societally-based 'co-evolution' of human consciousness, but only after the birth of the *H. sapiens* species [91],[186],[203]. This relatively, high rate of evolution through *societal-based 'co-evolution'* in comparison with the rather slow, preceding biological evolution, is consistent with consciousness 'co-evolving' rapidly as the result of primitive societal interactions [186] that have acted as a powerful, and seemingly essential, 'driving force', 'catalyst' or stimulus.

Nevertheless, time intervals of accelerated biological evolution are likely to have occurred repeatedly, depending not only upon environmental changes but also on the positioning of such organisms on the epigenetic landscape, relative to the location of basins of dynamic or 'strange' attractors. Note here also that the possibility of a first migration of *H. sapiens* from Africa to the Middle East between 200,000 and 100,000 B.C. has also been suggested by paleoanthropologists.

On the other hand, one may expect that the degree of complexity of human primitive societies which supported and promoted the emergence of human consciousness was also significantly higher than those of hominin bands characterized by what one might call individual *hominin 'quasi–consciousness'*. It would seem that the passage of the threshold towards human consciousness and awareness of the human self may associated with the ascent of the *Cro–Magnon* man, which is thought to belong to the modern species of *Homo* *sapiens sapiens*, (chromosomally descended from the Y haplogroup F/mt haplogroup N populations of the Middle East). This important transition seems to have taken place between 60,000 and 30,000 years ago through the formation of Cro–Magnon, primitive human 'societies'–perhaps consisting of small bands of 16 to 25 individuals, or so, sharing their hunting, stone tools, wooden or stone weapons, a fire, the cooked food, a cave, one large territory, and ultimately reaching human consensus and self-awareness.

After human consciousness has fully emerged along with complex social interactions within pre–historic *H. sapiens* tribes, it is likely to have also acted as a positive feedback on both the human individual and society development through multiple social interactions, thus leading to an ever increasing complexity of the already ultra-complex system of the human mind. Subsequently, it became possible to form the first historic human societies which have emerged some 10,000 years ago. As in the case of the primordial, the question is raised if *H. sapiens* might have evolved in different places at different times, and it is often answered in the negative, thus supporting uniqueness.

The claim is defended here that the emergence of ultra-complexity required the occurrence of '*symmetry breaking*' at several levels of underlying organization, thus leading to the unique *asymmetry* of the human brain–both functional and anatomical; such recurring symmetry breaking may also require a sharp complexity increase in our representations of mathematical-relational structure of the human brain, and also of human consciousness. Arguably, such repeated symmetry breaking does result in *layered complexity dynamic patterns* [40], [210] in the human mind that appear to be organized in a hierarchical manner. Thus, 'conscious planes' and the focus of attention in the human mind [188],[190] are linked to an emergent *context-dependent variable topology* of the human brain, which is most evident during the brain's developmental stages guided by environmental stimuli such as human/social interactions; the earliest stages of a typical human child's brain development would be thus greatly influenced by its mother.

6.3. *Memory and the Emergence of Consciousness*

Although the precise nature of human memory is unknown one may hypothesize that it involves processes that induce and regulate, or control the formation of higher levels of memory accessible to consciousness from the culmination of those at lower stages that may not be accessible to the conscious mind. Just as chemical reactions and syntheses engage canonical functors to build up neural networks [14],[23], and natural transformations between them can enable 'continuous' perceptions, the various neural dynamic super-network structures– at increasingly higher levels of complexity– may support the dynamic emergence of the *continuous, coherent and* global *'flow of human consciousness'* as a new, *ultra-complex level of the mind*–as clearly distinct from, but also linked to– the underlying human brain's localized neurophysiological processes. Clearly, however, human consciousness without memory and a perception of both time and space is virtually impossible, but the reverse for memory may not necessarily hold true, as even a single neuron retains at least a transient 'memory' of the most recent history of its stimuli.

6.4. *Local-to-Global Relations: A Higher Dimensional Algebra of Hierarchical Space/Time Models in Neurosciences. Higher-Order Relations (HORs) in Neurosciences and Mathematics.*

The Greeks devised *the axiomatic method*, but thought of it in a different manner to that we do today. One can imagine that the way Euclid's Geometry evolved was simply through the delivering of a course covering the established facts of the time. In delivering such a course, it is natural to formalize the starting points, and so arranging a sensible structure. These starting points came to be called *postulates, definitions and axioms*, and they were thought to deal with real, or even ideal, objects, named points, lines, distance and so on. The modern view, initiated by the discovery of non–Euclidean geometry, is that the words points, lines, etc. should be taken as undefined terms, and that axioms give the *relations* between these. This allows the axioms to apply to many other instances, and has led to the power of modern geometry and algebra. Clarifying the meaning to be ascribed to 'concept', 'percept', 'thought', 'emotion', etc., and above all the *relations* between these words, is clearly a fundamental but time–consuming step. Although relations–in their turn–can be, and were, defined in terms of sets, their axiomatic/categorical introduction greatly expands their range of applicability well–beyond that of set-relations. Ultimately, one deals with *relations among relations* and relations of higher order. We are thus considering here the possibility of a novel higher-dimensional algebra approach to spacetime ontology and also to the dynamics of the human brain and the meta–level of the human mind. The human brain is perhaps one of the most complex systems –a part of the human organism which has evolved about two million years ago as a separate species from those of earlier *hominins/hominides*. Linked to this apparently unique evolutionary step– the evolution of the *H. sapiens* species– human consciousness emerged and co-evolved through *social* interactions, elaborate *speech*, *symbolic communication/language* somewhere between the last 2.2 million and 60,000 years ago. The oldest remains of *H.sapiens* in Europe date back to 46,000 BC, and are interestingly intermingled with those of Neanderthals. We shall thus consider in our essay the dynamic links between the biological, mental and social levels of reality. The most important claim defended here is that the *ultra-complex* process of processes (or meta-process) usually described as *human consciousness* is correlated with certain functions of fundamentally *asymmetric* structures in the human brain and their corresponding, recursively non–computable dynamics/psychological processes. These are *non-commutative* dynamic patterns of structure-function and can be therefore represented by a Higher Dimensional Algebra of neurons, neuronal (both intra- and inter-) signaling pathways, and especially high-level psychological processes viewed as *non-computable patterns* of linked-super-aggregate processes of processes,...,of still further sub-processes. Therefore, a local-to-global approach to Neural Dynamics and the human brain functions seems to be necessary based upon the essential dynamic relations that occur between the hierarchical layers of neural structures and functions in the brain; the emphasis here will be primarily on the human brain functions/biodynamics. We shall consider certain essential relations in Neurosciences and Mathematics as a potential starting point for a Categorical Ontology of Neurosciences. We conclude here that contrary to previous philosophical and ontological thinking, *low-level* relations are quite *insufficient* to define or understand consciousness, which is intrinsically based on meta–level, ***higher order relations (HORs)***, such as those involved in meta–processes of processes. Rather than being 'immaterial', the mind's meta-level works through such HORs, thus subsuming the lower order relations and processes to do its bidding without any need for either 'mystical'/'spiritualistic' pseudo–explanations or an equally baffling/inconceivable (human) mind–brain split with no physical connections between them. This extremely important theme will be further discussed in the remaining sections.

6.5. *What is Consciousness?*

The problem of how the human mind and brain are related/correlated with each other has indeed many facets, and it can be approached from many different starting points. Herbert Spencer in 1898 [249] simply 'defined' consciousness in a very broad sense as a ***relation*** between a 'subject' and an 'object'. The problem is, of course, that of defining the subject– a definition that needs to be, at least in part, *self-referential* [237]–[238], and thus beyond the confines of Boolean logic, but now still approachable *via* Quine's logic. Over the last twenty five years considerable attention has been paid to the question of whether or not mental processes have some physical content, and if not, how do they affect physical processes. It would seem however that previously not all the 'right', or key, questions have been asked about human consciousness. We have seen in the previous subsection that the meta–level question can be answered in the context of consciousness by HORs; Spencer's vague idea of a simpler, lower relation is insufficient here because of the general/fundamental asymmetry or distinction between 'object' and 'subject': an external object can often be defined in terms of simpler relations than those of the meta–level of the 'subject'. On the other hand, when the human mind becomes itself the 'object' of study by the 'subject', both are characterized by (albeit different) *meta–level* relations, and one also needs to consider then the *next higher order relations* (NHORs) between such meta-level relations. (As in Category Theory, simple morphisms are insufficient; the 'raison d' être' of mathematical categories are the *natural transformations*/functorial morphisms between functors, which as explained above are defined only on the second order meta–level, and thus involve NHORs.) Awareness, or self–consciousness, would then *a fortiori* involve such NHORs. Thus, both consciousness of others and the consciousness of one's self involve such ultra–complex NHOR's that are part and parcel of HDA; as we shall see later, the consciousness of others developed first through primitive human, social (tribal) interactions,followed by self-consciousness on the same ultra–complex level of reality. As we shall see, this view is consistent with both recent philosophical psychology and with sociological enquiries into primitive *H. sapiens* tribes. In this monograph we shall not attempt to debate if other species are capable ofr consciousness, or to what extent, but focus instead on the ultra-complex problems raised by human consciousness and its co-emergence, as well as co-evolution, with human society.

The nature of thought is the subject of psychology and its related fields. One area of psychology–cognitive psychology–uses information processing as a framework for understanding the human mind. More generally, psychology studies also: perception, learning, problem solving, memory, attention, language and emotion, etc., by a great variety of investigative methodologies. Thus, historically, the leading disciplines concerned with the human mind have been philosophy and psychology, that were later joined also by behavioral science, cognitive science, logics, biomathematics, neuroscience and neural net computing. In addition, the physics of complex systems and quantum physics have produced stimulating discussions on the nature of consciousness. On the other hand, the study of neural networks and their relation to the operation of single neurons can profit a great deal from complex systems dynamic approaches. There is, however, no substantial, experimental evidence that quantum processes in the brain are *directly* correlated with any mental activity. One also has to pose here the related important question–as Deacon [91] did: *why don't animals have language?* Some mammals, for example, may show good evidence of intelligence in many other respects, yet fluent, symbolic language with meaning is altogether beyond their abilities. Parrots can learn only to repeat, but not generate meaningful, short sentences. Deacon also examined what it is unique about the human brain that makes it capable of *symbolic speech with meaning.* Unlike, Mumford [195], Deacon [91] seems to have missed the important point of the rhythmic dances and symbolic rituals in primitive human societies as the turning point for ordering and training the emerging human mind coupled to an orderly society in which reification has most likely played also the key role in the further co–evolution/advancement of the mind, the language and the human society. This latter, 'magic' triangle was not considered by Deacon; he only considered the human brain $\rightleftarrows$ language co–evolution, and did not seem to appreciate the role(s) played by the 'primitive' human societies in the development of the unique human mind and consciousness; here the adjective 'primitive' is employed in the historical sense of *pre-historic*, or pre-dating human civilizations history that began about 10,000 years ago.

Attempting to define the human mind and human consciousness run into similar problems to those encountered in attempting to define Life; there is a long list of attributes of human consciousness from which one must decide which ones are the essential ones and which ones are derived from the primary attributes. Human consciousness is *unique*– it does not share its essential attributes with any other species on Earth. It is also unique to each human being even though, in this case, certain 'consensual'/essential attributes do exist, such as, for example, *reification*, and we shall return to this concept later in this section. Defining the human mind –whether in terms of simpler concepts than the mind itself or in abstract terms– encounters major difficulties mainly associated with the practical impossibility of its direct observation or experimentation; there remains, however, the possibility of defining the human mind as reflected by its creations, or 'products', such as: syntactic speech, writing, logic, problem solving, information exchange and storage capabilities, as well as its many other facets studied through the investigative methodologies of experimental and cognitive psychology. Thus, we know of no other species capable of writing, and therefore capable of transmitting information and the acquired knowledge/data from generation to generation; this also means that the societies of the present are generally built upon the experiences accumulated from those of the past, perhaps with the notable gaps caused by the loss of the ancient library of Alexandria, or the debated loss of the empire/land of Atlantis. Certain philosophers divide human consciousness into *phenomenal consciousness*, which is *the experience itself* of humans, and *access consciousness*, which is the processing of the things/items from the experience. There remains however the ontic gap between phenomenal and access consciousness–the memory storage of information, thoughts, past experiences, etc., for example– which, in itself, is an integral part of human consciousness and also serves as an essential link between the two, along with the *awareness of continuous time* throughout the conscious human life. Thus, William James in his popular "Principles of Psychology" [149] considered human consciousness as *"the stream of thought"* that never returns to the same exact 'state'. Both *continuity* and *irreversibility* are thus claimed as key, defining attributes of consciousness. We note here that our earlier metaphor for evolution in terms of 'chains of local (mathematical) procedures' may be viewed from a different viewpoint in the context of human consciousness–that of chains of 'local' thought processes leading to global processes of processes..., thus emerging as a 'higher dimensional' stream of consciousness. Moreover, in the monistic –rather than dualist–view of ancient Taoism the individual flow of consciousness and the flow of all life are at every instant of time interpenetrating one another; then, Tao in motion is constantly *reversing* itself, with the result that consciousness is *cyclic*, so that everything is –at some point– without fail changing into its opposite. One can visualize this cyclic patterns of Tao as another realization of the Rosetta biogroupoids that we introduced earlier in a different context– relating the self of others to one's own self. Furthermore, we can utilize our previous metaphor of 'chains of local procedures' –which was depicted in Figure 1–to represent here the "flow of all life" (according to Tao for example) not only in biological evolution, but also in the case of the generic local processes involving sensation, perception, logical/'active' thinking and/or meditation that are part of the 'stream of consciousness' (as described above in dualist terms). There is a significant amount of empirical evidence from image persistence and complementary color tests in perception for the existence of such cyclic patterns as those invoked by Tao and pictorially represented by the Rosetta biogroupoids in Figure 2; this could also provide a precise representation of the ancient Chinese concept of "Wu-wei" –literally 'inward quietness'–the perpetual changing of the stream of both consciousness and the unconscious into one another/each other. 'Wu', in this context, is just awareness with no conceptual thinking. Related teachings by Hui-neng can be interpreted as implying that *"consciousness of what is normally unconscious causes* both *the unconscious and consciousness to change/become something else than what they were before".*

The important point made here is that there is a very wide spread of philosophical approaches, ranging from the Western duality to the 'neutral monistic' (Spencerian), and the Eastern (monistic) views of Consciousness and Life. On the other hand, neither the Western nor the Eastern approaches discussed here represent the only existing views of human consciousness, or even consciousness in general. The Western 'science' of consciousness is divided among several schools of thought: *cognitive psychology*–the mainstream of academic orientation, the *interpretive psychoanalytic tradition*, *the 'humanistic' movement*, and finally, the *trans–personal psychology* which focuses on practices towards 'transcendence' in the sense of 'beyond consciousness', rather than with the Kantian meaning of 'beyond phenomenal experience'.

Therefore, our novel approach to human consciousness involving the ontological theory of levels, the emergence of ultra-complexity and meta-levels, as well as the highly complex relations underlying its various functions, differs quite significantly from both psychological and philosophical theories of consciousness by attempting to construct a categorical and HDA framework of consciousness which is both relational and non-Abelian in nature. Moreover, our extended Topos concept involving many-valued logic also allows the consideration of nuances of thought, intuition, relations undelying emotions, as well as the implication or involvement of variable topological and algebraic structures during the emergence of human consciousness, human development, learning and anticipation processes that are severely constrained either by Boolean logic or the standard topos with a Heyting (i.e.) commutative) logic classifier.

6.6. *The Emergence of Human Consciousness as an Ultra-Complex, Meta–⟨System⟩ of Processes and Sub-processes.*

The *ultra–complexity level* is defined in our essay as *the human mind's meta–level*, or the mental level, which comprises certain, unique dynamic patterns; it is conceived as *meta–process* of layered sub–processes, emerging to the most complex level of reality known thus far to man (considered as 'the mind–subject' observing other 'minds–objects'). This meta-level emerges from and interacts with the super–complex activities and the higher level processes that occur in special, super-complex subsystems of the human brain; such brain, or neural processes that were discussed in the previous section seem to be coupled through certain synergistic and/or mimetic interactions in human societies. In this sense, we are proposing a non–reductionist, categorical ontology that possesses both universal attributes and a top level of complexity encompassed only by human consciousness. However, several species seem also to possess subject awareness even though the individual nature of awareness differs dramatically *de facto* from that of *H. sapiens.* Whereas states of the mind, intention, qualia etc. are ingredient factors of consciousness that instantaneously occur with subjective awareness, none of these seem to be essential for the latter. Bogen discusses in [55] the neurophysiological aspect of *awareness* in relationship to the intra–laminar nuclei (ILN) which is a critical site when normal consciousness is impaired as the result of thalamic injury. However, his conclusions remain so far as speculative as many other so–called 'mechanisms' of consciousness.

As a working hypothesis, we propose here a provisional, and quite likely incomplete, definition of human consciousness as an *ultra–complex* process integrating numerous super-complex 'sub-processes' in the human brain that are leading to a *'higher-dimensional ontological, mental level'* capable of: 'free will', new problem solving, and also capable of speech, logical thinking, generating new conceptual, abstract, emotional, etc., ontological structures, including –but not limited to–'awareness', self, high-level intuitive thinking, creativity, sympathy, empathy, and a wide variety of 'spiritual' or 'mental' *introspective* experiences. It may be possible to formulate a more concise definition but for operational and modelling purposes this will suffice, at least provisionally. The qualifier *'ultra-complex'* is mandatory and indicates that the ontological level of consciousness, or mental activities that occur in the conscious, '(psychological) state', is *higher* than the levels of the underlying, *super-complex* neurodynamic sub-processes leading to, and supporting, consciousness. On this view, although the mental level cannot exist independently without, or be existentially separated from the neurodynamics, it is nevertheless distinct from the latter. This looks like a Boolean logic paradox which is avoided if one considers human consciousness and/or the mind as a **meta**–⟨**system**⟩ of intertwined mental and neurodynamic processes; such a meta-<system> would have no boundary in the sense described in Section 3, but a horizon. This proposed solution of the 'hard problem' of psychology is neither dualistic (i.e., Cartesian) nor monistic –as in Taoism or Buddhism; our novel view simply disagrees in detail with Descartes' dualism, Buddhist monism, and also with materialism that assumes only one ontic level–that of matter, as it is an anti-thesis of *"tertium non datur"*– the excluded third possibility, simply because reality is likely to be much more complex than crysippian/ Boolean logic, as Hegel– as well as Buddhist philosophers– were very fond of repeatedly and correctly pointing out. It is also consistent with Kant's warnings in his critique of pure reason and his findings/logical proofs of formally undecidable propositions that preceded by three centuries Gödel's theorem (restricted to the incompleteness of arithmetics). Clearly, self-representation, self-awareness and the origin of symbolic meaning/semantics in general is resolved without any of the Russellian paradoxes of type as the meta–system has a different essence and existence than the various systems of processes from which it emerged; one is therefore obliged to consider the ultra–complex, ontology level, a **meta–level** of existence.

It is argued in [70] that the brain cannot be expected to work in a 'linear' or 'serial' fashion, and that some form of 'higher dimensional algebra' will be needed to model how the brain works, with its multi-level structure, and distributed mode of communication among its parts. The work of Ehresmann and Vanbremeersch in [103,104] also discusses how categories endowed with colimits, can model many aspects of organisations. It will be interesting to see how that work can be combined with the models coming from higher dimensional algebra, particularly in representing the topology of the human brain network processes (occurring in the two interconnected brain hemispheres) that underlie and support consciousness [33]. In order to obtain a sharper, more 'realistic' (or should one perhaps say instead, 'ideal') representation of consciousness one needs consider psychological 'states' (Ψ), 'structures' (Φ) as well as consciousness modes (CMs) in addition, or in relation to neurophysiological network structure and neural network super-complex dynamics. According to James [149]-[150] consciousness consists in a *'continuous stream or flow'* of psychological 'states' which never repeats the same 'state' because it is continually changing through the interaction with the outer world, as well as through internal thought processes (suggested to have been metaphorically expressed by the saying of Heraclitus that *'one never steps in the same water of a flowing river'*, and also by his *"Panta rhei"–"Everything flows!"*). However, the recurrence of patterns of thoughts, ideas, mental 'images', as well as the need for *coherence of thought*, does seem to establish certain psychological 'states' (Ψ), psychological 'structures' (Φ), and indeed at least two 'modes' of consciousness: an active mode and a *'receptive', or 'meditative'* one. Whereas the 'active' mode would be involved in biological survival, motor, speech/language, abstract thinking, space or time perception and volitional acts (that might be localized in the left-side hemisphere for right-handed people), the 'receptive' mode would be involved in muscle-or general-relaxation, meditation, imagination, intuition, introspection, and so on (i.e., mental processes that do not require interaction with the outside world, and that might be localized in the right-side cerebral hemisphere in right-handed people). The related issue of the obvious presence of two functional hemispheres in the human brain has been the subject of substantial controversy concerning the possible dominance of the left-side brain over the right-side, as well as the possibility of a subject's survival with just one of his/her brain's hemisphere. All such related 'psi' categories and attributes are relevant to a mathematical representation of consciousness as an ultra-complex, meta–process emerging through the integration of super-complex sub-processes or layers.

Fundamental ontology research into the nature of Life and Consciousness should be of very high priority to society in view of their importance for every human being. Clearly, a thorough understanding of how complex levels emerge, develop, and evolve to still higher complexity is a prerequisite for making any significant progress in understanding the human brain and the mind. Categorical Ontology and HDA are tools indeed equal to this hard task of intelligent and efficient learning about our own self, and also without straying into either a forest of irrelevant reductionist concepts or simply into Platonic meditation. Thus, such approaches and tools may not be enough for 'all' future, but it is one big, first step on the long road of still higher complexities.

6.7. *Intentionality, Mental Representations and Intuition.*

We present here a concise summary of three essential mental processes, the first and second groups of processes being essential to the existence of human consciousness, and the third–that of intuition– seemingly key to human creativity beyond Boolean logic and step-by-step, 2-valued logic inferences. Although these cannot be at all separated from memory except in a formal sense, we are considering memory in a separate section as in the first instance the human mind retains and 'filters' representations of perceptions; obviously, the mind also memorizes ideas, concepts, elaborate mental constructs, etc. in addition to images, sounds, sensations, and so on. Furthermore, the physical basis, or supporting biophysical/neural processes of sensations and perceptions is much better understood than that of memory, or the other three key mental processes considered next.

6.7.1. *Intentionality*

Consciousness is always *intentional*, in the sense that it is always directed towards (or intends) ***objects*** [149]-[150]. Amongst the earlier theories of consciousness that have endured are the *objective self-awareness* theory and Mead's *psychology of self-consciousness* [190]. According to the pronouncement of William James in 1890 (pp.272-273 in ref.[149]),

> *"the consciousness of objects must come first".*

The reality of everyday human experience 'appears already objectified' in consciousness, in the sense that it is 'constituted by an '*ordering of objects*' (*lattice*) which have already been designated 'as objects' before being reflected in one's consciousness. All individuals that are endowed with consciousness live within a web, or *dynamic network*, of human relationships that are expressed through language and symbols as *meaningful objects*. One notes in this context the great emphasis placed on *objects* by such theories of consciousness, and also the need for utilizing '*concrete categories that have objects with structure'* in order to lend precision to fundamental psychological concepts and utilize powerful categorical/ mathematical tools to improve our representations of consciousness. A new field of categorical psychology may seem to be initiated by investigating the categorical ontology of ultra-complex systems; this is a field that might possibly link neurosciences closer to psychology, as well as human ontogeny and phylogeny. On the other hand, it may also lead to the 'inner', or *'immanent', logics* of human consciousness in its variety of forms, modalities (such as 'altered states of consciousness'-ACS) and cultures.

Furthermore, consciousness classifies different objects to different 'spheres' of reality, and is capable also of moving through such different spheres of reality. The world as 'reflected' by consciousness consists of multiple 'realities'. As one's mind moves from one reality to another the transition is experienced as a kind of 'shock', caused by the shift in attentiveness brought about by the transition. Therefore, one can attempt to represent such different 'spheres of reality' in terms of concrete categories of objects with structure, and also represent the dynamics of consciousness in terms of families of categories/'spheres of reality' indexed by time, thus allowing 'transitions between spheres of reality' to be represented by functors of such categories and their natural transformations for 'transitions between lower-order transitions'. Thus, in this context also one finds the need for categorical colimits representing coherent thoughts which assemble different spheres of reality (*objects reflected* *in consciousness*). There is also a common, or *universal, intentional character of consciousness.* Related to this, is *the apprehension of human phenomena as if they were '**things**'*, which psychologists call *'reification'*. Reification can also be described as the extreme step in the process of objectivation at which the objectivated world loses its comprehensibility as an enterprise originated and established by human beings. Complex theoretical systems can be considered as reifications, but "*reification also exists in the consciousness of the man in the street*" [181]. Both psychological and ethnological data seem to indicate that the original apprehension of the social world (including society) is *highly reified* both ontogenetically and philogenetically.

Kant [155] considered that the internal structure of reasoning, or the 'pure reason', was essential to human nature for knowledge of the world, but the inexactness of empirical science amounted to limitations on the overall comprehension. At the same time, in his 'critique of the pure reason' Kant warned that transcedental ideas can be neither proven nor disproven as they cannot be phenomenally checked or validated. Brentano considered intentional states as defined via the mental representation of objects regulated by mental axioms of reason. As it is experienced, Freeman [111] regards intentionality as the dynamical representation of animal and human behaviour with the aim of achieving a particular state circumstance in a sense both in unity and entirety. This may be more loosely coined as 'aboutness', 'goal seeking' and or 'wound healing'.

6.7.2. *Mental Representations- The Hypothesis of a < System > of Internal Representations in Psychology and Cognitive Sciences.*

Mental representations are often considered in psychology and cognitive sciences (including neocognitivism, cf. Dennett, 1981 [92]) as fundamental; the concept has been therefore intensely debated by philosophers of psychology, as well as psychologists, and/or cognitive scientists. The following discussion of such concepts does not imply our endorsement of any of such possible philosophical interpretations even though it is hard to see how their consideration and the mental roles they play could be either completely or justifiably avoided. The important question of how *language-like* are mental representations is one that is often debated by philosophers of the mind.

According to Harman, "thought may be regarded as consisting in large part of *operations on 'sentences under analysis'* [136]. However, Harman [136], and also Fodor [110], claim that only some mental representations are highly language-like, and that not all of them are such. Brentano's position regarding *intentionality* of mental representations was clearly stated as making the distinction between the physical and mental realms. Other philosophers are less supportive of this view; a cogent presentation of various positions adopted by philosophers of the mind vis a vis mental representations was provided by Field (Ch.5 in ref.[54]). As pointed out by Field [109], postulating the *irreducibility* of mental properties (e.g., to physical or neurophysiological ones) raises two main problems: the problem of *experiential* properties and the problem of *intentionality* raised by Brentano. Most mental properties, if not all, seem to be *relational* in nature; some for example may relate a person, or people, to certain items called "propositions" that are usually assumed not to be linguistic. Field claims however that in order to develop a psychological theory of beliefs and desires one could avoid propositions altogether and utilize "something more accessible" that he calls *sentences.* Thus, mental representations would be expressed as relations between people and 'sentences' instead of propositions. Unlike propositions then, sentences do have linguistic character, such as both syntax and semantics, or else they are sentence-analogs with significant grammatical structure, perhaps following Tarski's compositional theory. On the other hand, Harman is quite critical of those compositional semantics that regard a *knowledge* of truth-conditions as what is essential in semantics (... '*'Davidson's theory would be circular*"). Furthermore, Gilbert Harman wrote: "no reason has been given for a compositional theory of meaning for whatever system of representation we think in, be it Mentalese or English" [136]. Then, *"organisms which are sufficiently complicated for the notions of belief and desire to be clearly applicable have* ***systems of internal representations*** *(SIR) in which sentence-analogs have significant grammatical structure*", writes Field. On this hypothesis of SIR, a belief involves a ***relation between organisms and sentence-analogs in a SIR*** for organisms of 'sufficient complexity'. From a functionalism standpoint which abstracts out the physical structure of particular organisms, the problem arises how psychological properties are realized by such organisms, as well as the questions of how to define a *realization* of a psychological property, and how to define "what a psychological property itself is". Therefore, *"if you do not construe belief relationally, you need a physical realization of the belief relation"* (p.91 of [109]).

6.7.3. *Intuition.*

There is much that can be said about intuition in a logical or mathematical sense; this precise meaning of intuition is further addressed in ref.[37] where the necessary, logical and mathematical concepts are also available in a rigorous form. In this section, we shall however consider the broader meaning of intuition, as it seems to play a major role in developing new concepts, theories, or even paradigm shifts. One may speak of intuition correlating to some form of intentionality which momentarily may not be derivable to a semantic/linguistic meaning regardless of a causal framework but may involve some kind of 'pictorial analogy'. Perhaps this is relevant to the sign language of the deaf, which is three–dimensional and contains semantic elements. But intuition may also involve nuances of learning and wording towards boundaries within the overlaps of 'fuzzy nets' which, as we propose, are based on the principles of non-commutative (multi-valued) n- Łukasiewicz logics (cf.[32],[118]-[120]). Ultimately, if an intuition is 'correct' or 'wrong' in the 'collective eyes of society', is determined through an *objectivation* process which pervades all human culture: it is either accepted or rejected by an intellectual majority in a specific human society. As this process is rarely based only on logic, and may also involve experiential considerations, objectivation does not have the 'permanent' character that this word may imply. Paradigm shifts in science are, in this sense, major re-considerations of objectivation of scientific concepts and theories. A remarkable paradigm shifts and re-objectivations seems to be now occurring in the ontology of higher complexity systems and processes, currently labelled as 'Complexity Theory' or 'Complex Systems Biology' (when the latter is restricted to living organisms).

An '*intuitive space*' or *intuition* **layer** of complexity (cf. Poli in [208]; Baiaπu and Poli, 2010 [40]) might thus appear to exist apart from, or relatively independent of, how experiences can be rationalized. Since intuition is a property attributed to the human mind (or to the 'autobiographical self' in the sense of Damasio [89], it has therefore to be considered as conceptually different from 'instincts' or brain-initiated reflexes. In keeping with the above considerations, human 'intuition' may thus be regarded as a by–product of an ultra–complex 'system' of processes occurring in the unique human mind, an essential intrinsic attribute, of that 'system' of processes.

6.8. *Propositional Attitudes*

Following Fodor [110] *propositional attitudes* are assumed to ascribe or represent *relations between organisms and internal representations* (p.45 in [110]). Furthermore, they seem to be often identified with the inner speech and/or thought. According to Fodor cognitive psychology is a revival of the representational 'theory' of the mind: "*the mind is conceived as an organ whose function is the manipulation of representations, and these in turn, provide the domain of mental processes and the (immediate) objects of mental states*" [110].

If mental representations, on the other hand, were to require the existence of an 'observer' or 'exempt internal agent' that can interpret what is being represented, one would face an infinite regress. Therefore, the claim was made that the human mind's representations related to the thinking process and/or human solving/cognition processes are in fact $\langle$ *representations* $\rangle$ *of representations*, or even some kind of 'self-representation'. In this respect also, the human mind is *unique* by comparison with that of any lower animal, if the latter can be at all considered as a 'mind' because it clearly has only limiting boundaries and no conceivable horizon. Note the critique of the propositional attitude concept by Field in the previous subsection, and the latter's hypothesis that *sentence–analogs* in a SIR can replace propositional attitudes in psychology. The difference between the two views seems to lie in the specific nature of propositional attitudes (that may be somewhat intangible) and sentence-analogs in an SIR that may be 'tangible' in the sense of having significant grammatical structure (syntax, semantics, etc.), e.g., being more language-like. Furthermore, as attitudes are intentionality related the propositional *attitudes* may be more complex and richer than Field's sentence–analogs. One also notes that Rudolf Carnap suggested in [81] that propositional attitudes might be construed as *relations between people and sentences* they are disposed to utter. The reader may also note that in these two subsections, as well as in the next one, the emphasis is on the role of *relations* and properties–instead of objects–in the philosophy of psychology, and thus a categorical, logico-mathematical approach to SIR seems to be here fully warranted, perhaps including a Tarskian compositional semantics, but with Harman's critical *proviso* and warnings cited above!

Either representational 'theory', or hypothesis, leaves open the questions:

1. What relates internal representations to the outside world?, and

2. How is SIR semantically interpreted? or How does one give meaning to the system of internal representations?

Perhaps Field's proposal [109] could be implemented along the Tarskian compositional semantics in a many-valued setting, such as the Łukasiewicz generalized topos (LGT), that was first introduced in [32],[37], that can provide an adequate conceptual framework for such semantic interpretations *with nuances specified by many truth values* instead of a single logical value.

6.9. *Psychological Time, Spatial Perceptions, Memory and Anticipation*

Subdivisions of space and spatiotemporal recognition cannot satisfactorily answer the questions pertaining to the brains capability to register qualia–like senses arising from representations alone (such as a sense of depth, ambiguity, incongruity, etc.) Graphic art in its many forms such as cubism, surrealism, etc. which toy around with spatial concepts, affords a range of mysterious visual phenomena often escaping a precise neuro–cognitive explanation. For instance, we can be aware of how an extra dimension (three) can be perceived and analyzed from a lower dimensional (respectively, two) dimensional representation by techniques of perceptual projection and stereoscopic vision, and likewise in the observation of holographic images [197]. Thus, any further analysis or subdivision of the perceived space would solely be a task for the 'minds–eye' (see Velmans, 2000, Ch.6 in ref.[261] for a related discussion). Through such kaleidoscopes of cognition, the induced mental states, having no specified location, may escape a unique descriptive (spatiotemporal) category. Some exception may be granted to the creation of holographic images as explained in terms of radiation and interference patterns [197], but still the perceived three dimensional image is *illusory* since it depends on an observer and a light source; the former then peers into an 'artificial' space which otherwise would not have existed. However, the concept of holography heralds in one other example of the ontological significance between spacetime and spectra in terms of a fundamental duality. The major mathematical concept for this analysis involves the methods of *the Fourier transform* that decompose spatiotemporal patterns into a configuration of representations of many different, single frequency oscillations by which means the pattern can be re–constructed *via* either summation or integration. Note, however, that visualizing a 4-dimensional space from a picture or painting, computer-generated drawing, etc., is not readily achieved possibly because the human mind has no direct perception of *spacetime*, having achieved separate perceptions of 3D-space and time; it has been even suggested that the human brain's left-hemisphere perceives time as related to actions, for example, whereas the right-hemisphere is involved in spatial perception, as supported by several split-brain and ACS tests. This may also imply that in all other species–which unlike man– have symmetric brain hemispheres temporal perception–*if it does exist at all, which is doubtful*– is not readily separated from space perception, at least not in terms of localization in one or the other brain hemisphere.

The mathematical basis relating to the topographical ideas of Pribram's models [218] lies in part within the theory of harmonic analysis and (Lie) transformation groups. Relevant then are the concepts of (Lie) groupoids and their convolution algebras/algebroids together with species of 'localized' groupoids. Variable groupoids (with respect to time) seem then to be relevant, and thus more generally is the concept of a fibration of groupoids (see, e.g.[140]) as a structural descriptive mechanism. Such observations, in principle representative of the ontological theory of levels, can be reasonably seen as contributing to a synthetic methodology for which psychological categories may be posited as complementary to physical, spatiotemporal categories. Such theories as those of Pribram [218],[197] do not fully address the question of universal versus personal mind: how, for instance, does mind evolve out of spatiotemporal awareness of which the latter may by continuously fed back into the former by cognition alone? The answer –not provided by Pribram [218], but by previous work carried out by Mead (cca.1850) in [190]–seems to be negative because human consciousness appears to have evolved through social, consensual communications that established symbolic language, self-talk and thinking leading to consciousness, as modelled above by the Rosetta biogroupoid of human/hominin social interactions. A possible, partial mechanism may have involved the stimulation of forming an increased number of specialized 'mirror neurons' [200] that would have facilitated human consciousness and symbolism through the evoked potentials of mirror neuron networks; yet another is the *synaesthesia*, presumably occurring in the Wernicke area (W) of the left-brain, coupled to the 'mimetic mirror neurons' thus facilitating the establishment of permanent language centers (Broca) linked to the W-area, and then strongly re-enforced and developed through repeated consensual social human interactions. In the beginning, such interactions may have involved orderly rituals and ritual, 'primitive' dances whose repetitive motions and sensory perception acts may have enforced collectively an orderly 'state' in the primitive *Homo*'s minds. Such periodic and prolonged rituals in primitive societies–as suggested by Mumford [195] – may have served the role of ordering the mind, *prior to*, and also facilitating, the emergence of human speech! Thus a collective system of internal representations and reification in the human mind may have had its very origin in the primitive rituals and ritualistic dancing prior to the development of truly human speech. The periodic, repetitive action of ritual dancing, charged with emotional content and intentionality, may have served as a very effective *training* means in such primitive tribal societies, much the same way as human champions train today by rhythmic repetition in various sports. Clearly, *both a positive feedback*, and a *feedforward (anticipatory)* mechanism were required and involved in the full development of human consciousness, and may still be involved even today in the human child's mind development and its later growth to full adult consciousness. Interestingly, even today, in certain tribes the grandfather trains the one-year old child to 'dance' thus speeding up the child's learning of speech. One can consider such observations as contributing substantially towards a resolution of the *'hard problem'* of consciousness: how can one fully comprehend the emergence of *non–spatial* forms arising from one that is *spatial* (such as the brain) within the subjective manifold of human sensibility? The functional brain matter is insentient and does not by itself explain causal, spatiotemporal events as agents of consciousness. However, there have been attempts as for example those made by Austin in 1998 to 'link' the brain's neurobiology with the mind in order to explain the qualities of conscious experience [8], in this case within a Buddhist-philosophical (strictly *non-dual* or *monistic*) context of awareness; the latter is inconsistent with the Western, *dual* approach extensively discussed in this essay, in the sense of the mind vs. the brain, organism vs. life, living systems *vs* inanimate ones, super-complex *vs* simple systems, environment *vs* system, boundary *vs* horizon, and so on, considering them all as pairs of *distinct* (and *dual/apposed*, but not opposed) ontological items. Surprisingly, reductionism shares with Buddhism a *monistic* view of the world–but coming from the other, physical extreme– and unlike Buddhism, it reduces all science to simple dynamic systems and all cognition to mechanisms.

The questions of mind–brain 'interface' remain largely unanswered as there have been very few determined attempts at even posing correctly such questions, and even fewer at seriously investigating how the mind correlates with observable brain processes (for example through MRI, SQUID magnetometry, NIR/laser fluoresacence, PET scanning, etc. measurements on conscious *vs* unconscious human brains combined with detailed psychological studies). Whereas Kantian intuitionism seems to reduce matters to an interplay of intellect and imagination as far as differing qualities of 'space' are concerned, the dictum of physics claims without failure *'non–existence if it can't be measured'*.

There are several philosophers who have made the claim of metaphysical limits upon intellectually conceived representations, to the extent that definitive explanations might remain beyond the grasp of human comprehension (e.g., Kant in 1778 [155], and also McGinn in 1995 [188]). Quine accepted that analytic statements are those that are true by definition, but then he successfully argued that the notion of truth by definition was unsatisfactory. Others (cf. Bennett and Hacker in ref.[51]) in part echoing Gilbert Ryle's pronouncement of "categorical problems" [240]–in the philosophical sense (i.e., categorial)– argue that brain science alone cannot explain consciousness owing to a plague of intrinsic (metaphysical–categorial) errors such as when a certain neuropsychological entity is conceived as a 'linear' superposition of its constituent parts (cf 'the mereological fallacy'); in this regard, Bennett and Hacker [51] spare no reductionist so-called 'theories of neuroscience'.

Even though the human brain consists in a very large (approximately 100,000,000,000), yet finite, number of neurons– and also a much higher number of neuronal connections greater than 10^{29}– the power of thought enables conscious humans to construct symbols of things, or items, *apart from the things themselves*, thus allowing for our extension of representations to higher dimensions, to infinity, enlightenment, and so on, paradoxically extending the abilities of human consciousness very far beyond the apparent, finite limitations, or boundaries, of our super-complex, unique human brain. One notes here also that the psychological concept of dynamic 'net without boundary' occurring and moving in the 'conscious plane', but often with a specific focus [188] , leads to a 'completely open', variable topology of the human mind. Thus, one may not be able to consider the human mind as a 'system' because it seems to possess no boundary– but as an *'open multiverse of many layers, or super-patterns of processes of processes,... with a horizon'.* The mind has thus freed itself of the real constraints of spacetime by separating, and also 'evading', through virtual constructs the concepts of time and space that are being divided in order to be conquered by the human free will. Among such powerful, 'virtual' constructs of the human mind(s) are: symbolic representations, the infinity concept, continuity, evolution, multi-dimensional spaces, universal objects, mathematical categories and abstract structures of relations among relations, to still higher dimensions, many-valued logics, local-to-global procedures, colimits/limits, Fourier transforms, and so on, it would appear without end. This view of the human mind seems consistent with the proposal made by Gregory Bateson [45]-[48], who put forward an interesting scheme of "*logical levels of meaning*", and went on to emphasize that the human 'mind is *not confined* to the body but ramifies out *informationally* into the symbolic universe around it.', i.e., the human mind alone has a horizon, not a strict, or fixed, boundary. Bateson also argued that the 'ecology of mind' is an *ecology of pattern, information, and ideas* embodied in things that are material forms. Thus, a science which would limit itself to counting and weighing such embodiments would only arrive at a very distorted understanding of the mind. Gregory Bateson characterized what he meant by a *mind* (or *mental 'system'*) in his "*Pathologies of Epistemology.*"(on p.482), where a mental 'system' was defined as one with a capacity to process and respond to information in a *self–corrective* or *autopoietic* manner, just as it is the characteristic of living systems from cells to forests, and from primitive society to human civilizations. Then, he also developed such a characterization into a list of *defining criteria* for the human mind; in his view, the mind is composed of multiple material parts whose arrangements allow for *both process and pattern.* Upon this view, the human mind is **not separable** from its material base and the traditional Cartesian dualism separating the mind from the body, or the mind from matter, is considered erroneous; a 'mind'–in this extended Batesonian (but not Leibnitz-like) sense–

can thus also include non–living components as well as multiple organisms; it may function for either brief or extended periods, and *is not necessarily defined by a boundary*, such as an enveloping skin or the skull. For Bateson, however, consciousness– if present at all– is always *only partial*. This emphasis on mental 'systems' as "including more than single organisms" leads Gregory Bateson to insisting that the *unit* of survival is always *the organism and its environment*. Furthermore, Bateson elaborates the notion that in the world of mental processes, the *difference* is the analog of ***cause*** (the " *difference that makes a difference*"), and then argues that embedded and interacting systems have a capacity to select a pattern, or patterns, from apparently random elements, as it happens in both evolution and learning; he calls the latter "two great stochastic processes." Interestingly, he was also able to explore the way in which such an analogy underlies all the *"patterns which connect"*. Then, Bateson develops a typology of habitual errors in the ways of thinking, some that are only minor, and some that are potentially lethal [45]-[48]. Although the human mind is able to conceive higher dimensions and infinity, it may also lead through the wrong political decisions to the total destruction of life and consciousness on earth–as in a nuclear 'accident', or through intentional conflagration and environmental destruction. This moral and societal 'duality'– as long as it persists– may make to us, all, the difference between the continued existence of human society and its irreversible disappearance on earth. As an informational related cause, Bateson for example traced the origin of *destructive* human actions to *inappropriate descriptions*, and also argued that '*'what we believe ourselves to be should be compatible with what we believe of the world around us,"*[48]; yet, knowledge and belief do involve deep chasms of ignorance or unknowing. Bateson was thus convinced that human society should have a "respect for the systemic integrity of nature, in which all plants, animals and humans alike, *are part of each other's environment*", albeit as *unequal* partners.

7. Emergence of Organization in Human Society. Social Interactions and Memes

We shall consider first an emergent human pre-historic society and

then proceed to examine the roles played by social interactions and memes generated by society. Finally, we shall consider the potential dangers of arbitrary political decision–making that could lead to accidental but permanent extinction of both human civilization and all life on earth.

7.1. *A Rosetta Biogroupoid of Social, Mutual Interactions: The Emergence of Self and Memes through Social Interactions*

One may consider first a human pre-historic society consisting of several individuals engaged in hunting and afterwards sharing their cooked food around a fire. The ability to share food as an interlude to extensive social interactions and exchanges seems to be unique to humans, perhaps because of the pre-requisite *consensual* interactions, which in their turn will require similar mental abilities, as well as an understanding of the need for such sharing in order to increase the survival chances of each individual.

It seems that the awareness of the self of the other individuals developed at first, and then, through an extension of the concept of others' self to oneself, *self awareness* emerges in a final step. Such pre–historic societal interactions that are based on consensus, and are thus mutual, lead to a natural representation of the formation of 'self' in terms of a *'Rosetta* *biogroupoid'* structure as depicted below, but possibly with as many as twenty five branches from the center, reference individual:

$$\begin{array}{ccccc} & & \text{Neighbour's Self} & & \\ & & \updownarrow & & \\ \text{Neighbour's Self} & \longleftrightarrow & \text{Oneself} & \longleftrightarrow & \text{Neighbour's Self} \\ & & \updownarrow & & \\ & & \text{Neighbour's Self} & & \end{array} \tag{0.5}$$

Diagram (0.6): *A Rosetta biogroupoid* of consensual, societal interactions leading to self-awareness, one's self and full consciousness; there could be between 4 to 24, or more individuals in a pre-historic society of humans; here only four are represented as branches.

One may consider modern society as a second order meta-level of the human organism, with the ultra–complex system of the human mind, as its first order meta–level. The overall effect of the emergence of the unique, *ultra-complex human mind meta–level* and the co-evolution of human society has been the complete and uncontested *dominance* by man of all the other species on earth. Is it possible that the emergence of the highly complex society of modern man is also resulting in the eventual, complete domination of man as an individual by 'his' highly complex society? The historical events of the last two centuries would seem to be consistent with this possibility, without however providing certainty of such an undesirable result. However, ontological theory of levels considerations seem to exclude such a possibility as the resulting (hypothetical, 'first-order meta-level' society would be non-generic and thus unstable). Furthermore, as we have seen that society has strongly influenced human consciousness, indeed making possible its very emergence, what major effect(s) may the modern, highly complex society have on human consciousness? Or, is it that the biological (evolutionary) limitations of the human brain are preventing, or partially 'filtering out' the complexification pressed onto man by the highly–complex modern societies? There are already existing arguments that human consciousness has already changed since ancient Greece, but has it substantially changed since the beginnings of the industrial revolution? There are indications of human consciousness perhaps 'resisting'– in spite of societal reification–changes imposed from the outside, perhaps as a result of *self-preservation of the self*. Hopefully, an improved complexity/super- and ultra–complexity levels theory, as well as a better understanding of spacetime ontology in both human biology and society, will provide answers to such difficult and important questions.

7.2. *Social Interactions and Memes*

Our previous discussion concerning the ontology of biological and genetic networks has a counterpart in the social networks, related to how scientific technologies, socio–political systems and cultural trademarks comprise the methodology of the planet's evolutionary development (or possibly its eventual demise!). Thus, Dawkins coined in 1982 the term *'meme'* as a unit of cultural information having a societal effect, and that is either imitated, or *replicated* by society members, groups or organizations, or that is passed from one person to another by *non-genetic* means (as by imitation) [90]. Indeed, the origin of the word 'meme' appears to have been the Greek word *"mimetismos"* that means 'something which is being imitated'. Thus, memes are artificially, not naturally created, although some memes may strive hard to appear 'natural', or are even claimed to be such. As an example, going to an opera, a clasical concert, or an industry job interview, all participants–including the performers– know that the accepted norm, or the meme, is that one must "dress to impress", among other things. Such memes can also be transmitted, and thus be 'inherited' by different societies throughout history. In philosophy, a *meme is a unit of a packet of information*, such as, the unit of cultural ideas, practices or symbols that is assimilated by one mind from another through various means such as gestures, rituals, speech and/or writing, including the Internet; an *Internet meme* is thus a concept, image, video, catchphrase, etc, that spreads quickly from one person to another via the Internet. TV and film-propagated memes are somewhat similar to Internet ones, but are more tightly controlled or even censored. News headlines may also rapidly spread memes around the globe, being like the internet one of the major means for globalising memes. Internet memes like the " Rick Roll" ,"Lolcats" and various animated gifs abound. Forwarded emails may be a very effective means of spreading memes to the point of becoming annoying, and it is considered rude to either send auto-DMs (direct messages) on a regular basis, or capitalize words in emails and posted sentences on a blog–the written equivalent on the web of shouting. Many internet memes "offer either a funny or an entertaining reprieve from the daily grind". Advanced technology has markedly increased both the informational content and the rate of transmission of memes, thereby increasing tremendously their impact on society. The providers of such rapid transmission means of information and modalities for propagating memes are then very substantially rewarded by the society at large. At the other end of the 'spectrum of memes' are inked tattoos, such as those from African tribes, American indian tribes, LA Ink, etc., that brought a shared or common identity to the members of the tribe by inscribing the meme identifying the tribe on the skin of all individual tribe members. Certain memes, such as unspoken 'rules' of human behavior, may be tough to grasp because they are mostly unspoken and enforced only by peer pressure; the main purpose of such memes appears to serve as an *'internal boundary'* within the society system, separating the 'insiders' from the 'outsiders' of a cultural group, by telling people who is new and fluent in the social 'environment', as distinct from individuals who are different from the 'accepted norm'–whatever that may be. Such memes tell the modern 'tribe' who belongs to it, and who does not yet belong, or may not belong. Therefore, such memes can also be considered as a general means of *classification* within society, in the ontic sense rather than the rigid social classes that were, and still are, strictly enforced throughout history.

Many memes, and indeed large parts of the cultures present, for example, in the Western societies, can be traced back to the medieval times, and even further back to the Roman Empire and the preceding Greek antiquity. Interestingly, most of the antique Egyptian, Alexandrian, Aztec, or even Mongolian memes seem to have been however lost throughout the rest of the world, perhaps along also with a very small group of genes present in the populations that generated them; nevertheless, many of the memes related to such ancient religions and myths have been absorbed into the cultures of modern societies, albeit in modified forms. The persistence and transmission of memes depends, at least in part, on a process of *objectivisation* that filters out in the long term those memes that clash with culture and tradition, or may be considered as unacceptable by society.

Although it has been suggested that memes have 'hereditary' characteristics similar to how the human form, certain behaviours, instincts, etc. are indeed genetically inherited, in fact, the nature of memes and their underlying operational logic is quite different from the multi-valued logic of genetic newtworks. Thus, it is generally, and trivially, accepted that intentionality has no bearing on, or connection with, the genetic transmission of hereditary characteristics, or phenotypes. Clearly, memetic characteristics are quite distinct from their genetic counterparts. Cultures evolve through levels and species compete. Memetic 'competition' can be found in the conflicting ideologies of opposing political camps who defend their policies in terms of economics, societal needs, employment, healthcare, and so on. It is not however 'competition for survival' as it cannot be said in the strict sense that memes are 'alive'. It could also be argued that the meme's informational level belongs in fact to a *meta-level* relative to that of the mind of each individual, even though it has emerged from the latter as a concept or pattern of information which is being shared by many minds. Thus, Csikzentmihalyi suggested in [87] a definition of a meme as *"any permanent pattern of matter or information produced by an act of human intentionality"*; thus, a meme is a concept auxiliary to that of the ontology of a 'level': to an extent, the latter is the result of generations of a 'memetic evolution' via the context of their ancestry. Memes may be initiated as the result of a neuro–cognitive reaction to stimuli and its subsequent assimilation in an effective communicable form, but their establishment *requires intenionality* which does occur at the meta-level of human consciousness, not merely at the neurophysiological level, as well as its propagation throughout a human population or group, thus occurring at the meta-level of many minds that share the same meme. Any type of invention, including the scientific one, no matter how primitive, would seem to satisfy these general criteria. Once a meme is generated there is a subsequent *inter–reaction* with its inventor, and also with the people who strive to develop and use it, and so forth (e.g. from the first four–stroke combustion engine to the present day global automobile industry). Csikzentmihalyi suggested in ref.[87] that mankind is not as much threatened by natural biological evolution as by the overall potential content of memes, and by their uncontrolled spreading and development. This is actually straightforward to see as global warming and environmental pollution by man may both serve as striking examples of such widely spread memes, especially in societies with dictatorial political systems. Moreover, the more technologically advanced a society may be the greater is the danger posed by memes which spread uncontrollably and rapidly to the global level. Thus, whether we consider the memes in terms of weapons, aeronautics, whatever, their 'destination' reaches as far as mankind can exploit it; those who are likely to benefit most are the founding fathers of new industrial cultures, inventors and explorers alike, the reformers of political and educational systems, and so on. Unfortunately, memes can generate their own (memetic) 'disorders', such as addiction, obesity and pollution that may reach *unacceptably high levels for sustainability* of the human society as a whole. Thus, to a large extent, the meta-level of the human memetic systems are patently complex and therefore extremely difficult to control; they may also be regarded as ontologically different sublevels of the society's meta–level, each such sublevel possessing its own respective characteristic order of causality.

Related to memetic and autopoietic systems are those of *social prosthetic systems* in which the limitations of the individual cognitive capacity can be extended via participation within varieties of socio–environmental networks. Loosely speaking, the mind 'uses' the world and 'enduring relationships" as extensions of itself. As for many of the highly complex systems considered in this essay, the underlying structures can be represented in terms of equivalence classes, thus leading to configurations of either Rosetta groupoids of social interactions, and/or to the more complex groupoid atlas structures.

The theme of *Biomimetics*, unlike that of memes, is concerned with mimicking attributes and properties of biological networks, as well as with attempting to imitate human problem solving and human intelligence.

7.3. *The Human Use of Human Beings. Political Decision Making*

In his widely-read books on Cybernetics and Society [269], Norbert Wiener–the founder of Cybernetics– attempted to reconcile mechanistic views and machine control concepts with the dynamics of modern society. He also advocated the representation of living organisms in terms of *variable* machines or variable automata (formally introduced in [14], [22]). As discussed in previous sections, the variable topology is a far richer and extremely flexible structure, or system of structures, by comparison with the rigid, semigroup structure of any machine's state space. Thus, a variable topology dynamics provides a greatly improved metaphor for the dynamic 'state spaces' of living organisms which have emerged as super-complex systems precisely because of their variable topology. Many other society 'evolution' issues, and well-founded concerns about the human misuse of human beings, raised by Wiener are much amplified and further compounded today by major environmental issues. It remains to be seen if complexity theories will be able to fare better than Cybernetics in addressing *'the human use of human beings'* as Wiener has so aptly labelled the key problem of human societies, past and present. Wiener's serious concerns towards rigid and unjustified control of academic freedom through arbitrary political decisions by 'politically powerful' administration bureaucrats, as well as the repeated, gross misuses of scientific discoveries by politicians/dictators, etc., are even more justified today than half a century ago when he first expressed them; this is because the consequences of such severe controls of creative human minds by uncreative ones are always very grave indeed, in the sense of being extremely destructive. Thus, it is not the A– or H–/neutron bombs 'in themselves' that are extremely dangerous, but the political intent/potential, or actual decision to make and use them against human beings which is the culprit. Such considerations thus lead one into the subjects of ethics and morality, two very important philosophical/ontological fields that remain well beyond the horizon of our monograph.

8. Biomimetics, Cybernetics and the Design of Meta-Level Artificial Intelligence Systems

Biomimetics has a long history of attempted mimicking or imitation by man of animal life forms in his mechanical inventions. From musical boxes, cuckoo clocks and mechanical dancers or toys to modern robots there is a wide array of robotic devices that superficially mimic some animal or human actions, without however being endowed with any of the qualities that we assign to human intelligence. This theme does link up therefore with both the general science of controlling systems– Cybernetics, and with Computer Science and AI Engineering. Whereas one may consider a computer also to be a more elaborate– but still primitive form of imitation of the human brain– it cannot yet be claimed to exhibit something that would even remotely resemble human intelligence, chess-playing algorithms notwithstanding. At the other end of the spectrum of biomimetic inventions, so called 'neural' networks with artificial 'neurons" may claim to capture some of the learning potential of the human brain. Recently, attempts are being made to build much bigger networks with numbers of interacting units approaching that of the number of neurons in a human brain, in the strange belief that 'bigger is always better', forgetting of course the fate/ complete disappearance of the largest dinosaurs. Such a naive belief that complexity is only a matter of quantitative scaling, rather than topology and true dimensional scaling, is the result of common reductionist thinking taken to its extreme. There would be a very serious technological challenge to achieve the connectivity of a human brain that runs into trillions of connections, not to mention the programming and MV-logic challenges for such gigantic informational networks that would attempt to mimick the human brain. It is thus evident that in order to built an intelligent device the inventors themselves would have to be endowed with the level of intelligence needed to build such AI devices; that also means that in order to mimick the meta-levels on which human intelligence is based, and invent an advanced AI system closer to mimicking human intelligence, one would have to understand in some detail the process or processes involved in the emergence of the meta-level of human consciousness that entails intelligence as experienced by humans in society. Clearly, this also has an important training, or educational/learning aspect that is not a matter of mere reflexes, the number of existing neurons that are connected, or making new 'neural' connections. Thus, although autistic children have the same huge number of neurons as the non-autistic children do, whereas the latter are integrated with relative ease in the human society [196] and thus, educated, the former pose serious problems in that respect. An interesting hypothesis concerning autism is that in autistic subjects, either children or adults, the mirror neurons necessary for normal social communication, interactions, training and education are somehow deficient [200]. If this were even partially true, a way to increase the degree of 'intelligence' of an advanced AI system (AAI) would be to design it with built-in analogues of mirror neurons that would be suitable for interactions of the AAI system with human educators so that it would then become possible for specialized human teachers to educate an AAI and teach it how to solve new problems. That would be indeed a sign of recognizable intelligence for any AAI system, but there would be many people opposing the mere presentation of such a blueprint for AAI, and it could be outlawed in a manner similar to that of human cloning. On the other hand, such an AAI system– when used as a prosthetic device implanted in human patients– could help millions of autistic patients, as well as patients with Alzheimer disease.

Thus, taking further Norbert Wiener's idea of mimicking living organisms in terms of variable automata or machines in ref.[269], one can consider novel designs of AI 'systems' as variable automata (VAs) that interact/communicate with other, different VAs, in such a way that a meta-level emergence may occur to an AI super-system made of multiple VAs, akin to a supercomputer, S_c, made up from varying VA modules that adapt, anticipate, etc, mimicking autopoiesis in biological organisms or societies. From a formalization viewpoint this would require introducing some meta-level category of categories, or a *super-category.* An alternative approach is to join the VA modules according to new HDA rules for consistent super-computation at the meta-level or levels. This novel idea opens a field of Biomimetics aimed at designing more powerful AI meta- 'systems' (AIMs) than the Boolean logic-based designs of existing AI systems [1]-[265]. A direct approach to designing AIMs compatible with AIs would be to utilize the main result that has been already obtained for categories of LM-logic algebras in refs.[118]-[120] and [31]-[32]. Specifically, one can make use of the Fundamental Logic Adjointess Theorem recalled in ref.[120], and utilize the adjointess between the category of centered Łukasiewicz n-logic algebras, $Cluk_n$, and the category of Boolean logic algebras, **Bl**. Then, the left- and right- adjoint functors betwen these two categories of logic algebras allow one, at least in principle, to design AIMs with centered Łukasiewicz logic that are compatible with AIs based on simple Boolean logic algebras [163]. The natural equivalence classes defined by the adjointness relation in the Adjointness Theorem determine, or define a *logical groupoid structure* that is then computable.

Yet, a more complex AI multi-system, AI_m, than the AIMs could utilize LM-logic *algebroids* instead of LM-logic algebras, and the approach discussed above would still work for designing AI-compatible AI_ms operating with centered LM-*algebroids* instead of LM-algebras. (An *algebroid* being broadly defined here as 'an algebra with many objects'). This obviously may however involve somewhat tricky HDA developments.

A more difficult extension of this design approach to designing advanced AIMs involves the use of VAs that are *quantum* computers, because in this case the quantum LM-logic algebra, LQL, is generally *not centered*, and thus it is incompatible with any Boolean logic algebra or current AI systems. Here we have merely sketched however the potential of this approach for designing powerful quantum super- or meta- computers that are 'super-intelligent' by comparison with the existing AIs operating with Boolean logic algebras. For the current Boolean logic based designs of computers and AIs there is already a rapidly growing literature on standard Category Theory applications to programming and basic designs of such AIs (see for example refs.[1],[23],[194],[197],[201]). In ref. [151], for example, the use of *n-categories* and 'weak n-categories', and thus HDA concepts, were also considered in the context of computer science and novel AI designs.

Nevertheless, human fear of *super-intelligent* AI 'terminators' may very well act as a deterrent, or major obstacle to the development of such advanced AIMs. Other related issues to those addressed in this section were also discussed in more detail for both AI and Cognitive Science in refs.:[1],[194],[200]-[225],[252],[262].

9. Conclusions

A combined, novel approach by CT, AT, HDA and LM-logic algebra was presented for the study of fundamental relational structures and physiological functions present in (living) higher organisms, based on our recent work in this direction, refs.[11]-[40],[69], as well as the recent work of R. Brown and his coworkers [68],[70]-[74]. An attempt was also made here to present a concise tutorial for the meta-level, HDA concepts relevant to understanding the human mind and other intelligent, complex systems, such as advanced AI meta-systems, or AIMs. Our approach– as outlined in this monograph– is also relevant to Complex Systems Biology, as well as sociological and environmental, theoretical studies that require an understanding of ultra-complexity levels.

Current developments in the SpaceTime Ontology of complex, super-complex and ultra-complex systems were presented covering a very wide range of highly complex systems and processes, such as the human brain and neural network systems that are supporting processes underlying human perception, consciousness and logical/abstract thought. Mathematical generalisations, such as higher dimensional algebra/HDA, are concluded to be some of the essential, logical requirements of the unification between complex system and consciousness theories that can lead to a deeper understanding of man's own spacetime ontology, which is claimed here to be both *unique* and *universal*.

New areas of Categorical Ontology are also most likely to develop as a result of the recent paradigm shift towards non-Abelian theories. Such new areas would be related to recent developments in: non-Abelian Algebraic Topology [68], non-Abelian gauge theories of Quantum Gravity, non-Abelian Quantum Algebraic Topology and noncommutative Geometry, that were here briefly outlined in relation to spacetime ontology.

Contrary to Spencer's statements in 1898 [249], matter, space and time do have known, definite attributes, and so does indeed Spacetime–a concept introduced later by Einstein and Minkowsky through a logical/mathematical, rigorous *synthesis* of experimental results with critical thinking and the elimination of the 'ether'. One notes however that the current physical concept of *vacuum* is far from being just empty space. There is currently an overwhelming consensus that spacetime is *relative* as stated by Poincaré and Einstein, not the Newtonian absolute, even though it has an *objective existence* (consistent with Spencer's (1898) contention that the Absolute has no objective existence). Standard quantum theories, including the widely-accepted 'Standard Model' of physics, lack the definition of either a time or a spacetime operator, but does have a space operator. Prigogine's introduction of a microscopic time super–operator [219] seems to be only a partial solution to this problem in quantum theory that allows the consideration of *irreversible* processes without which Life and Consciousness would be impossible, but that ultimately result also in their inevitable global disorganization ('ageing') and demise; for example, Prigogine's time super–operator can be properly defined only for quantum systems with an *infinite* number of degrees of freedom. On the other hand, introducing a *spacetime super–operator* in quantum theory- *à* la Prigogine's microscopic time super–operator - generates its own new series of problems, and of course, there is no such operator/super–operator defined in either Einstein's GR/SR or Newtonian mechanics. As complex, super– and ultra–complex dynamics is defined in essence by *irreversible* processes evolving in spacetime, which are the result of a multitude of quantum interactions and processes, the understanding and rigorous treatment of highly–complex systems is also affected by the limitations of current quantum theories; some of these current quantum–theoretical limitations in attempted applications to living organisms have been already pointed out by Baianu et al in [25]-[39]. In three related papers [36],[37] and [69], we have also considered further spacetime ontology developments in the context of Astrophysics, and also introduced novel representations of the Universe in terms of quantum algebraic topology and quantum gravity approaches based upon the theory of categories, functors, natural transformations, quantum logics, non-Abelian Algebraic Topology and Higher Dimensional Algebra; these approaches were then integrated with the viewpoint of Quantum Logics as part of a Generalised 'Topos'–a new concept that ties in closely Q-logics with many-valued, LM-logics and category theory. The latter synthesis may have consequences as important as the joining of space and time in the fundamental concept of spacetime modified by matter and energy.

The main results presented in this monograph are as follows:

• In Categorical Spacetime Ontology, the fundamental *relations and structure*, have a *non-commutative*/non-Abelian, fundamentally 'asymmetric' character of both top and bottom levels of reality; this is the origin of a paradigm shift towards non-Abelian theories in science, and of the need for developing a *non-Abelian Categorical Ontology*, especially as a complete, non-commutative theory of levels founded in LM– and Q– logics. The potential now exists for exact, symbolic calculation of the non-commutative invariants of spacetime through logical or mathematical, precise language tools (categories of LM–logic algebras, generalized LM–toposes, HHSvKT, Higher Dimensional Algebra, ETAS, and so on).

• The existence of *super-complex* systems in the form found in living organisms and their component biosystems is the result of highly-complex emergence and evolution processes that involved a *dynamic symmetry breaking* beginning at the molecular/quantum level and continuing to the higher levels of biological organization; succinctly stated:

no symmetry breaking and no emergence $\Longrightarrow$ ***no real complexity***.

• Human consciousness is operationally defined from the informational/cognitive psychology, simplifying viewpoint as:

Phenomenal consciousness $\Longrightarrow$ **Memory/time** $\Longrightarrow$ *Access consciousness*,

or

Experience $\Longrightarrow$ **Memory storage–time axis** &*processing the experience.*

Whereas such a reduction of human consciousness to the simplified- informational and operational definition provides access to information science concepts, it does not take into account the relational and functional ultra-complexity of the human mind, nor does it qualify for its inclusion in the ontological meta-level of the mental realm.

• The co-emergence and co-evolution of the unique human mind and society, with the emergence of an ultra–complex level of reality can be understood in terms of the emergence of human consciousness through co-evolution/societal interactions and highly efficient communication through elaborate speech and symbols. Following a detailed analysis, the claim is defended here that the human mind is more like a *'multiverse with a horizon, or horizons'* rather than merely a *'super-complex system with a finite boundary'*.

• There is an urgent need for *a resolution of the moral duality* between creation/creativity and the serious destruction threats posed to the human mind and the current society/civilization which is potentially capable of not only self-improvement and progress, but also of total Biosphere annihilation on land, in oceans, seas and atmosphere. The latter, dismal alternative would mean the *complete, rapid and irrevocable* reversal of several billion years of evolution–a total and permanent destruction of all life on planet Earth rather than only a mere, temporary involution. Arguably, the human minds and society may soon reach a critical and unique cross-road—with the nature of a potentially non-generic/strange dynamic attractor–unparalleled since the emergence of the first (andso humble) living primordial(s) on Earth.

Moreover, we have derived here several important consequences of non-commutative complex dynamics for human society and the Biosphere; potential non-Abelian tools and theories that are most likely to enable solutions to such ultra-complex problems were also pointed out in connection with the latter consequences. We have thus considered in this tutorial paper a very wide range of important problems whose eventual solutions require an improved understanding of the ontology of both the space and time (spacetime) dimensions of 'objective' reality, especially from both the relational complexity and universality/categorical viewpoints. Rapid progress through fundamental, cognitive research of Life and Human Consciousness that employs highly efficient, non-commutative tools, and/or precise 'language' is of greatest importance to human society, and to its continued survival and progress. Such progress necessarily leads to the development of a complete Categorical Ontology Theory of Levels and Emergent Complexity in HDA representations. Even though we have built a strong case for a Non-Abelian Categorical Ontology, and have also pointed out the major limitations of computers in simulating the biodynamics of entire organisms that are recursively non-computable, both commutative diagrams and recursive computations by digital computers are quite useful in providing partial analyses of subsystems dynamics of functional organisms; such digital computer simulations as suggested in [23] and [28] should be carried out by bearing in mind the intrinsic limitations of the computer simulations at the final conclusions stage of the analysis. In view of the increasing use of Abelian CT in both computer science and classification of experimental data by computers for ontological, as well as practical, purposes, one may expect a rapid expansion of CT and toposes to Categorical Ontology applications, albeit in its *commutative*, and thus in its more restrictive, constrained, or symmetric form. The first, as well as some of the subsequent, applications of CT in biology were also of the latter kind, i.e., Abelian: [16],[70],[74],[103]-[104],[124],[230],[235]-[236],[264], as there were some of its earlier applications to general quantum problems and quantum gravity [78]-[79],[202].

We have been unable however to cover in this monograph in any significant detail the broader, and very interesting implications of *objectivation* processes for human societies, cultures and civilizations. Furthermore, there are several possible extensions of our approach to investigating globally the *biosphere*.

Biosphere $\Longleftrightarrow$ Environment interactions remain therefore as a further object of study in need of developing a formal definition of the horizon concept, only briefly touched upon here and in ref.[40], also in this volume.

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1 Appendix: Background and Concept Definitions

1.1 Background to Category Theory

1.1.1 Categories

A *category* C consists of :

1. a class $\mathrm{Ob}(\mathsf{C})$ called the *objects of* C ;

2. for each pair of objects a, b of $\mathrm{Ob}(\mathsf{C})$, a set of *arrows* or *morphisms* $f : a \longrightarrow b$. We sometimes denote this set by $\mathrm{Hom}_{\mathsf{C}}(a, b)$. Here we say that a is the *domain of* f, denoted $a = dom\ f$, and b is the *codomain of* f, denoted $b = cod\ f$;

3. given two arrows $f : a \longrightarrow b$ and $g : b \longrightarrow c$ with $dom\ g = cod\ f$, there exists a composite arrow $g \circ f : a \longrightarrow c$.

Further

i) Composition is *associative* : given $f \in \mathrm{Hom}_{\mathsf{C}}(a, b), g \in \mathrm{Hom}_{\mathsf{C}}(b, c)$ and $h \in \mathrm{Hom}_{\mathsf{C}}(c, d)$, we have $h \circ (g \circ f) = (h \circ g) \circ f$.

ii) Each object admits an identity arrow $\mathrm{id}_a : a \longrightarrow a$, where for all $f \in \mathrm{Hom}_{\mathsf{C}}(a, c)$ and all $g \in \mathrm{Hom}_{\mathsf{C}}(b, a)$, we have $f \circ \mathrm{id}_a = f$, and $\mathrm{id}_a \circ g = g$.

Typical examples of a category are :

$\mathsf{C} = \mathsf{Set}$ where the objects of Set are sets and the arrows are simply set maps.

$\mathsf{C} = \mathsf{Top}$ where the objects of Top are topological spaces and the set of arrows $\mathrm{Hom}_{\mathsf{Top}}(X, Y)$ is the set of all continuous maps $f : X \longrightarrow Y$ between objects X and Y, and where the composition law in Top is the composition of continuous functions.

$\mathsf{C} = \mathsf{Group}$ where the objects are groups and the arrows $f : G \longrightarrow H$ are group homomorphisms between groups G and H .

Observe that $\mathrm{Ob}(\mathsf{C})$ need not be a set. When it is we shall say that C is a *small category*.

For the purpose of semantic modeling, let us say that an object $\mathbf{i}$ in any category is said to be *initial* if for every object a, there is exactly one arrow $f : \mathbf{i} \longrightarrow a$, whereas an object $\mathbf{t}$ in any category is said to be *terminal* if for every object a, there is exactly one arrow $f : a \longrightarrow \mathbf{t}$. Any two initial (resp. terminal) objects can be shown to be isomorphic.

Corresponding to each category C, is its *opposite category* C^{op} obtained by reversing the arrows. Specifically, C^{op} has the same objects as C, but to each arrow $f : a \longrightarrow b$ in C, there corresponds an arrow $f^{-} : b \longrightarrow a$ in C^{op}, so that $f^{-} \circ g^{-}$ is defined once $g \circ f$ is defined, and so $f^{-} \circ g^{-} = (g \circ f)^{-}$.

Let Q and C be categories. We say that Q is a *subcategory* of C if

1. (inclusion of object sets) each object of Q is an object of C ;
2. (inclusion of arrow sets) for all objects a, b of Q, $\mathrm{Hom}_{\mathsf{Q}}(a, b) \subseteq \mathrm{Hom}_{\mathsf{C}}(a, b)$;
3. composition '$\circ$' is the same in both categories and the identity $\mathrm{id}_a : a \longrightarrow a$ in Q is the same as in C .

A morphism m with codomain x is called *monic* if for all objects y and pairs of morphisms $u, v : x \to y$, $um = vm$ implies $u = v$. One can then define a *subobject* of x as an equivalence class of monics. The category of sets has preferred monics, namely the inclusions of subsets.

Let C and Q be two categories. *A covariant functor* is a function $\mathbf{F} : \mathsf{Q} \longrightarrow \mathsf{C}$ satisfying :

1. for each object a of Q, there is an object $\mathbf{F}(a)$ of C ;
2. to each arrow $f \in \mathrm{Hom}_{\mathsf{Q}}(a, b)$, there is assigned an arrow $\mathbf{F}(f) : \mathbf{F}(a) \longrightarrow \mathbf{F}(b)$, such that $\mathbf{F}(\mathrm{id}_a) = \mathrm{id}_{\mathbf{F}(a)}$, and if $g \in \mathrm{Hom}_{\mathsf{C}}(b, c)$, then $\mathbf{F}(g \circ f) = \mathbf{F}(g) \circ \mathbf{F}(f)$.

Likewise one can define a *contravariant functor* by standard modifications to the previous definition: $\mathbf{F}(f) : \mathbf{F}(b) \longrightarrow \mathbf{F}(a)$, $\mathbf{F}(g \circ f) = \mathbf{F}(f) \circ \mathbf{F}(g)$, etc.

A basic example is the (covariant) *forgetful functor* $\mathbf{F} : \mathsf{Top} \longrightarrow \mathsf{Set}$, which for any topological space X, $\mathbf{F}(X)$ is just the underlying set, and for a continuous map f, $\mathbf{F}(f)$ is the corresponding set map.

1.2 Natural Transformations and Functorial Constructions in Categories

Categorical constructions make use of *functors* between categories as well as the higher order 'morphisms' between such functors called *natural transformations* that belong to a '*2-category*' (see for example Lawvere, 1966). Such constructions also pave the way to *Higher Dimensional Algebra* which will be introduced in the next section. Especially effective are the *functorial constructions* which employ the *'hom' functors* defined next; this construction will then allow one to prove a very useful categorical result–*the Yoneda–Grothendieck Lemma.*

Let C be any category and let X be an object of C. We denote by $h^X : \mathsf{C} \longrightarrow \mathsf{Set}$ the functor obtained as follows: for any $Y \in Ob(\mathsf{C})$ and any $f : X \longrightarrow Y$, $h^X(Y) = \mathrm{Hom}_{\mathsf{C}}(X, Y)$; if $g : Y \longrightarrow Y'$ is a morphism of C then $h^X(f) : \mathrm{Hom}_{\mathsf{C}}(X, Y) \longrightarrow \mathrm{Hom}_{\mathsf{C}}(X, Y')$ is the map $h^X(f)(g) = fg$. One can also denote h^X as $\mathrm{Hom}_{\mathsf{C}}(X, -)$. Let us define now the very important concept of *natural transformation* which was first introduced by Eilenberg and Mac Lane (1945). Let $X \in Ob(\mathsf{C})$ and let $F : \mathsf{C} \longrightarrow \mathsf{Set}$ be a covariant functor. Also, let $x \in F(X)$. We shall denote by $\eta_X : h^X \longrightarrow F$ the *natural transformation* (or *functorial morphism*) defined as follows: if $Y \in Ob(\mathsf{C})$ then $(\eta_x)_Y : h^X(Y) \longrightarrow F(Y)$ is the mapping defined by the equality $(\eta_x)_Y(f) = F(f)(x)$; furthermore, one imposes the (*commutativity*) or naturality conditions on the following diagram:

$$
\begin{array}{ccc}
F(X) & \xrightarrow{\eta_X} & F(Y) \\
{\scriptstyle F(f)}\downarrow & & \downarrow{\scriptstyle F(g)} \\
G(X) & \xrightarrow[\eta_Y]{} & G(Y)
\end{array}
\tag{1.1}
$$

Lemma 1.1. ***The Yoneda–Grothendieck Lemma.*** *Let $X \in Ob(\mathsf{C})$ and let $F : \mathsf{C} \longrightarrow \mathsf{Set}$ be a covariant functor. The assignment $x \in F(X) \mapsto \eta_x$ defines a bijection, or one–to–one correspondence, between the set $F(X)$ and the set of natural transformations (or functorial morphisms) from h^X to F.*

This important lemma can be interpreted as stating that any small and concrete category can be realized as a full subcategory of a family of 'sets with structure' and structure preserving families of functions between sets [213] (see also *Section 6* and the references cited therein for its applications to the construction of categories of genetic networks or (**M,R**)–systems). Note also that the Yoneda–Grothendieck Lemma was previously employed to construct *generalized* Metabolic–Replication, or (**M,R**)–Systems in refs.[16]–[17], which are categorical representations of the *simplest* models of enzymatic (metabolic) and genetic networks [230].

We shall illustrate in subsequent *Sections 4 to 7* several applications to bionetworks of another very important type of functorial construction which *preserves colimits* (and/or *limits*); this construction is only possible for those pairs of categories which exhibit certain important similarities represented by an *adjointness relation*. Therefore, *adjoint functor* pairs (Kan, 1958) are here defined with the aim of utilizing their properties in representing *similarities* between categories of bionetworks, as well as preserving, respectively, their limits and colimits.

Definition 1.1. Let us consider two covariant functors F and G between two categories C and C' arranged as follows:

$$
\mathsf{C} \xrightarrow{F} \mathsf{C}' \xrightarrow{G} \mathsf{C}
\tag{1.2}
$$

We shall define F to be a *left adjoint functor* of G, and we define G to be a *right adjoint functor* of F, if for any X an object of category C, and any object X' of C', there exists a *bijection*

$$t(X,X') : \mathrm{Hom}_{\mathsf{C}}(X, G(X')) \longrightarrow \mathrm{Hom}_{\mathsf{C}'}(F(X), X') \ ,$$

such that for any morphism $f : X \longrightarrow Y$ of C and morphism $f' : X' \longrightarrow Y'$ of C', the following diagrams of sets and canonically constructed mappings are *natural* (or *commutative*) :

$$\begin{array}{ccc}
\mathrm{Hom}_{\mathsf{C}}(Y, G(X')) & \xrightarrow{t(Y,X')} & \mathrm{Hom}_{\mathsf{C}'}(F(Y), X') \\
\Big\downarrow h_{G(X')}(f) & & \Big\downarrow h_{X'}(F(f)) \\
\mathrm{Hom}_{\mathsf{C}}(X, G(X')) & \xrightarrow{t(X,X')} & \mathrm{Hom}_{\mathsf{C}'}(F(X), X')
\end{array} \tag{1.3}$$

$$\begin{array}{ccc}
\mathrm{Hom}_{\mathsf{C}}(X, G(X')) & \xrightarrow{t(X,X')} & \mathrm{Hom}_{\mathsf{C}'}(F(X), X') \\
\Big\downarrow h_{G(X')}(f) & & \Big\downarrow h_{X'}(F(f)) \\
\mathrm{Hom}_{\mathsf{C}}(X, G(Y')) & \xrightarrow{t(X,Y')} & \mathrm{Hom}_{\mathsf{C}'}(F(X), Y')
\end{array} \tag{1.4}$$

In particular, we shall denote by $\eta_X : X \longrightarrow GF(X)$, the morphism $t^{-1}(X, F(X))(\mathbf{1}_{F(X)})$. Also, we shall denote by

$$\varepsilon_{X'} : FG(X') \longrightarrow X' \ ,$$

the morphism $\varepsilon(G(X'), X')(\mathbf{1}_{G(X')})$, (cited from N. Popescu (1975), on p.11 in ref.[213]).

One can easily verify that the following diagrams, which are canonically constructed, are also *natural* in C and C' for any morphism $f : X \longrightarrow Y$ in C, and for any morphism $f' : X' \longrightarrow Y'$ in C', respectively.

$$\begin{array}{ccc}
X & \xrightarrow{\eta_X} & GF(X) \\
\Big\downarrow f & & \Big\downarrow GF(f) \\
Y & \xrightarrow{\eta_Y} & GF(Y)
\end{array} \tag{1.5}$$

and

$$\begin{array}{ccc}
FG(X') & \xrightarrow{\varepsilon_{X'}} & X' \\
\Big\downarrow FG(f') & & \Big\downarrow GF(f) \\
FG(Y') & \xrightarrow{\varepsilon_{Y'}} & Y'
\end{array} \tag{1.6}$$

Such adjoint functors *commute*, respectively, with either *limits* or *colimits* as specified by the following theorem (Theorem 5.4 on p.17 of N. Popescu, 1975).

Theorem 1.1. *Given categories* C *and* D*, let* $F : \mathsf{C} \longrightarrow \mathsf{D}$ *be the left adjoint of the functor* $G : \mathsf{D} \to \mathsf{C}$*. Then, one has:*

(1) F commutes with the colimit in C *of any functor;*

(2)] G commutes with the limit in D *of any functor.*

One also has the following important theorem (Theorem 5.3 of N. Popescu (1975), on p. 13 in ref.[213]).

Theorem 1.2. *Let* $F : \mathsf{C} \longrightarrow \mathsf{C}'$ *be a covariant functor. The following assertions are equivalent :*

(1) F is full and faithful and any object X' *of* C' *is isomorphic to an object* $F(X)$*, with* X *being an object of* C*;*

(2) F is full and faithful, and has a full and faithful left adjoint;

(3) F is full and faithful, and has a full and faithful right adjoint.

Definition 1.2. Two categories C and C' will be called *equivalent* if there is a covariant functor $F : \mathsf{C} \longrightarrow \mathsf{C}'$ which satisfies any of the three assertions in Theorem 1.2. The functor F will be called an *equivalence* from C to C'.

1.3 Higher order categories and cobordism

In higher dimensional algebra the concept of a category generalizes to that of an n–category. We list here a short (but tentative) dictionary of analogies between general relativity theory (GR) and quantum theory (QT) :

(1) (GR) pairs of spatial $(n-1)$–manifolds (M_1, M_2) – (QT) assigned Hilbert spaces H_1, H_2, respectively

(2) (GR) cobordism leading to a spacetime n–manifold M – (QT) (unitary) operator $T : H_1 \longrightarrow H_2$

(3) (GR) composition of cobordisms – (QT) composition of operators

(4) (GR) identity cobordism – (QT) identity operator.

The next step is to re–phrase this interplay of ideas categorically. So let Hilb denote the category whose objects are Hilbert spaces H with arrows the bounded linear operators on H . Let nCob denote the category whose objects are $(n-1)$–dimensional manifolds as above, and whose arrows are cobordisms between objects. Next we define a functor

$$Z : \mathsf{nCob} \longrightarrow \mathsf{Hilb} , \tag{1.7}$$

which assigns to any $(n-1)$–manifold M_1, a Hilbert space of states $Z(H_1)$, and to any n–dimensional cobordism $M : M_1 \longrightarrow M_2$, a (bounded) linear operator $Z(M) : Z(M_1) \longrightarrow Z(M_2)$, satisfying :

i) given n–cobordisms $M : M_1 \longrightarrow M_2$ and $\check{M} : \check{M}_1 \longrightarrow \check{M}_2$, we have $Z(M\check{M}) = Z(\check{M})Z(M)$.

ii) $Z(\mathrm{id}_{M_1}) = \mathrm{id}_{Z(M_1)}$.

Observe that i) means the duration of time corresponding to the cobordism M followed by that of the cobordism $\check{M}$, is the same as the combined duration for that of $M, \check{M}$. Part ii) says that given there is no topology change in some duration of time, then there is no effect on the state of the universe. Since a TQFT omits local degrees of freedom, only a topology change influences a change in the universe. Such a theory necessitates development, on the one hand, the relationship between nCob and n–categories (cf Baez and Dolan 1995), and on the other, that of a (non-commutative) theory of presheaves of Hilbert spaces/C^*–algebras which can be fitted into some quantum logical mechanism. Further, there is a necessity to realize the Grothendieck (1971) idea of *fibrations of n–categories over n–categories* as a possible unifying model for these theories.

1.4 Heyting–Brouwer Intuitionistic Foundations of Categories and Toposes

1.4.1 Subobject Classifier and the notion of a Topos

One of our main interests is in the notion of *topos*, a special type of category for which several (equivalent) definitions can be found in the literature. An important standard example is the category of (pre) sheaves on a small category C. We will need an essential component of the topos concept called a *subobject classifier*. In order to motivate the discussion, suppose we take a set X and a subset $A \subseteq X$. A characteristic function $\chi_A : X \longrightarrow \{0, 1\}$ specifies 'truth values' in the sense that one defines

$$\chi_A(x) = \begin{cases} 1 \text{ if } x \in A \\ 0 \text{ if } x \neq A \ . \end{cases} \tag{1.8}$$

A topos C is required to possess an analog of the truth–value sets $\{0, 1\}$. In order to specify this particular property, we consider a category C with a covariant functor $\mathsf{C} \longrightarrow \mathsf{Set}$, called a *presheaf*. The collection of presheaves on C forms a category in its own right, once we have specified the arrows. If $\mathcal{E}$ and $\mathcal{F}$ are two presheaves, then an arrow is a natural transformation $N : \mathsf{C} \longrightarrow \mathcal{F}$, defined in the following way. Given $a \in \mathrm{Ob}(\mathsf{C})$ and $f \in \mathrm{Hom}_{\mathsf{C}}(a, b)$, then there is a family of maps $N_a : \mathcal{E}(a) \longrightarrow \mathcal{F}(a)$, such that the diagram

$$\begin{array}{ccc} \mathcal{E}(a) & \xrightarrow{\mathcal{E}(f)} & \mathcal{E}(b) \\ {\scriptstyle N_a}\downarrow & & \downarrow{\scriptstyle N_b} \\ \mathcal{F}(a) & \xrightarrow{\mathcal{F}(f)} & \mathcal{F}(b) \end{array} \tag{1.9}$$

commutes. Intuitively, an arrow between $\mathcal{E}$ and $\mathcal{F}$ serves to replicate $\mathcal{E}$ inside of $\mathcal{F}$.

Towards classifying subobjects we need the notion of a *sieve* on an object a of $\mathrm{Ob}(\mathsf{C})$. This is a collection S of arrows f in C such that if $f : a \longrightarrow b$ is in S and $g \in \mathrm{Hom}_{\mathsf{C}}(b, c)$ is any arrow, then the composition $f \circ g$ is in S .

We define a presheaf $\Omega : \mathsf{C} \longrightarrow \mathsf{Set}$, as follows. Let $a \in \mathrm{Ob}(\mathsf{C})$, then $\Omega(a)$ is defined as the set of all sieves on a . Given an arrow $f : a \longrightarrow b$, then $\Omega(f) : \Omega(a) \longrightarrow \Omega(b)$, is defined as

$$\Omega(f)(S) := \{g : b \longrightarrow c : g \circ f \in S\} \ , \tag{1.10}$$

for all $S \in \Omega(a)$. Let $\uparrow b$ denote the set of all arrows having domain the object b . We say that $\uparrow b$ is the *principal sieve on* b, and from the above definition, if $f : a \longrightarrow b$ is in S, then

$$\Omega(f)(S) = \{g : b \longrightarrow c : g \circ f \in S\} = \{g : b \longrightarrow c\} = \uparrow b \ . \tag{1.11}$$

Let us return for the moment to our motivation for defining Ω . The set of truth values $\{0, 1\}$ is itself a set and therefore an object in Set, furthermore, the set of subsets of a given set X corresponds to the set of characteristic functions χ_A as above. Likewise if C is a topos, Ω is an object of C, and there exists a bijective correspondence between subobjects of an object a and arrows $a \longrightarrow \Omega$, leading to the nomenclature *subobject classifier*. In this respect, a typical element of Ω relays a string of answers about the status of a given object in the topos. Furthermore, for a given object a, the set $\Omega(a)$ enjoys the structure of a Heyting algebra (a distributive lattice with null and unit elements, that is relatively complemented).

1.5 Groupoids

Recall that a groupoid G is a small category in which every morphism is an isomorphism; we denote the set of objects by $X = \mathrm{Ob}(\mathsf{G})$. One often writes G_x^y for the set of morphisms in G from x to y .

A *topological groupoid* is a groupoid internal to the category Top . More specifically this consists of a space G, a distinguished space $\mathsf{G}^{(0)} = \mathrm{Ob}(\mathsf{G}) \subset \mathsf{G}$, called *the space of objects* of G, together with maps

$$r, s \ : \ \mathsf{G} \underset{s}{\overset{r}{\rightrightarrows}} \mathsf{G}^{(0)} \tag{1.12}$$

called the *range* and *source maps* respectively, together with a law of composition

$$\circ \ : \ \mathsf{G}^{(2)} := \mathsf{G} \times_{\mathsf{G}^{(0)}} \mathsf{G} = \{ \ (\gamma_1, \gamma_2) \in \mathsf{G} \times \mathsf{G} \ : \ s(\gamma_1) = r(\gamma_2) \ \} \longrightarrow \mathsf{G} \ , \tag{1.13}$$

such that the following hold :

(1) $s(\gamma_1 \circ \gamma_2) = r(\gamma_2)$, $r(\gamma_1 \circ \gamma_2) = r(\gamma_1)$, for all $(\gamma_1, \gamma_2) \in \mathsf{G}^{(2)}$.

(2) $s(x) = r(x) = x$, for all $x \in \mathsf{G}^{(0)}$.

(3) $\gamma \circ s(\gamma) = \gamma$, $r(\gamma) \circ \gamma = \gamma$, for all $\gamma \in \mathsf{G}$.

(4) $(\gamma_1 \circ \gamma_2) \circ \gamma_3 = \gamma_1 \circ (\gamma_2 \circ \gamma_3)$.

(5) Each γ has a two–sided inverse γ^{-1} with $\gamma\gamma^{-1} = r(\gamma)$, $\gamma^{-1}\gamma = s(\gamma)$.

For $u \in \mathrm{Ob}(\mathsf{G})$, the space of arrows $u \longrightarrow u$ forms a group G_u, called the *isotropy group of* G *at* u.

1.6 The concept of a Groupoid Atlas

Motivation for the notion of groupoid atlas comes from considering families of group actions, in the first instance on the same set. As a notable instance, a subgroup H of a group G gives rise to a group action of H on G whose orbits are the cosets of H in G. However a common situation is to have more than one subgroup of G, and then the various actions of these subgroups on G are related to the actions of the intersections of the subgroups. This situation is handled by the notion of *Global Action*, as defined in Bak 2000. A *global action* $\mathcal{A}$ consists of the following data:

(a) an indexing set $\Psi_{\mathcal{A}}$ called *the coordinate system of* $\mathcal{A}$, together with a reflexive relation $\leqslant$ on $\Psi_{\mathcal{A}}$;

(b) a set $X_{\mathcal{A}}$ and a family of subsets $(X_{\mathcal{A}})_\alpha$ of $X_{\mathcal{A}}$ for $a \in \Psi_{\mathcal{A}}$;

(c) a family of group actions $(G_{\mathcal{A}})_\alpha \curvearrowright (X_{\mathcal{A}})_\alpha$, i.e. maps $(G_{\mathcal{A}})_\alpha \times (X_{\mathcal{A}})_\alpha \longrightarrow (X_{\mathcal{A}})_\alpha$, with the usual group action axioms, for all $a \in \Psi_{\mathcal{A}}$;

(d) For each pair $\alpha \leqslant \beta$ in $\Psi_{\mathcal{A}}$, a group homomorphism

$$(G_{\mathcal{A}})_{\alpha \leqslant \beta} \ : \ (G_{\mathcal{A}})_\alpha \longrightarrow (G_{\mathcal{A}})_\beta \ .$$

This data must satisfy the following axioms:

(a) If $\alpha \leqslant \beta$ in Ψ_A, then $(G_{\mathcal{A}})_\alpha$ leaves $(X_{\mathcal{A}})_\alpha \cap (X_{\mathcal{A}})_\beta$ invariant.

(b) For each pair $\alpha \leqslant \beta$, if $\sigma \in (G_{\mathcal{A}})_\alpha$, and $x \in (X_{\mathcal{A}})_\alpha \cap (X_{\mathcal{A}})_\beta$, then $\sigma x = (G_{\mathcal{A}})_{\alpha \leqslant \beta}(\sigma) x$.

The diagram $G_{\mathcal{A}} : \Psi_{\mathcal{A}} \longrightarrow \mathsf{Groups}$, is called the *global group of* $\mathcal{A}$, and the set $X_{\mathcal{A}}$ is called the *enveloping set* or the *underlying set of* $\mathcal{A}$.

Suppose we have a group action $G \curvearrowright X$. Then we have a category $\mathsf{Act}(G, X)$ with object set X and $G \times X$ its arrow set. It is straightforward to show that $\mathsf{Act}(G, X)$ is actually a groupoid (Bak et al., 2006). Effectively, given an arrow (g, x), we have source and target defined respectively by $s(g, x) = x$, and $t(g, x) = g \cdot x$, represented by $(g, x) : x \to g \cdot x$. The composition of (g, x) and (g', x') is defined when the target of (g, x) is the source of (g', x'), i.e. $x' = g \cdot x$. This yields a composition $(g'g, x)$ as shown in:

$$x \xrightarrow{(g,x)} g \cdot x \xrightarrow{(g',gx)} g'g \cdot x \tag{1.14}$$

We have an identity at x given by $(1, x)$, and for any element (g, x) its inverse is $(g^{-1}, g{\cdot}x)$. A key point in this construction is that the orbits of a group action then become the connected components of a groupoid. Also this enables relations with other uses of groupoids.

The above account motivates the following. A *groupoid atlas* $\mathcal{A}$ on a set $X_{\mathcal{A}}$ consists of a family of 'local groupoids' $(\mathsf{G}_{\mathcal{A}})$ defined with respective object sets $(X_{\mathcal{A}})_\alpha$ taken to be subsets of $X_{\mathcal{A}}$. These local groupoids are indexed by a set $\Psi_{\mathcal{A}}$, again called the *coordinate system of* $\mathcal{A}$ which is equipped with a reflexive relation denoted by $\leqslant$. This data is to satisfy the following conditions (Bak et al., 2006) :

(1) If $\alpha \leqslant \beta$ in $\Psi_{\mathcal{A}}$, then $(X_{\mathcal{A}})_\alpha \cap (X_{\mathcal{A}})_\beta$ is a union of components of $(\mathsf{G}_{\mathcal{A}})$, that is, if $x \in (X_{\mathcal{A}})_\alpha \cap (X_{\mathcal{A}})_\beta$ and $g \in (\mathsf{G}_{\mathcal{A}})_\alpha$ acts as $g : x \longrightarrow y$, then $y \in (X_{\mathcal{A}})_\alpha \cap (X_{\mathcal{A}})_\beta$.

(2) If $\alpha \leqslant \beta$ in $\Psi_{\mathcal{A}}$, then there is a groupoid morphism defined between the restrictions of the local groupoids to intersections

$$(\mathsf{G}_{\mathcal{A}})_\alpha | (X_{\mathcal{A}})_\alpha \cap (X_{\mathcal{A}})_\beta \longrightarrow (\mathsf{G}_{\mathcal{A}})_\beta | (X_{\mathcal{A}})_\alpha \cap (X_{\mathcal{A}})_\beta \ ,$$

and which is the identity morphism on objects.

1.7 Locally Lie Groupoids

We shall begin here with the important definition of the concept of a locally Lie groupoid. *A locally Lie groupoid* (Pradines, 1966; Aof and Brown, 1992) is a pair (G, W) consisting of a groupoid G with range and source maps denoted α, β respectively, (in keeping with the last quoted literature) together with a smooth manifold W, such that :

(1) $\mathrm{Ob}(\mathsf{G}) \subseteq \mathrm{W} \subseteq \mathsf{G}$.

(2) $W = W^{-1}$.

(2) The set $W_\delta = \{W \times_\alpha W\} \cap \delta^{-1}(W)$ is open in $W \times_\alpha W$ and the restriction to W_δ of the difference map $\delta : \mathsf{G} \times_\alpha \mathsf{G} \longrightarrow \mathsf{G}$ given by $(g,h) \mapsto gh^{-1}$, is smooth.

(3) The restriction to W of the maps α, β are smooth and (α, β, W) admits enough smooth admissible local sections.

(4) W generates G as a groupoid.

We have to explain more of these terms. A *smooth local admissible section* of (α, β, W) is a smooth function s from an open subset of U of $X = Ob(\mathsf{G})$ to W such that $\alpha s = 1_U$ and βs maps U diffeomorphically to its image which is open in X. It is such a smooth local admissible section which is thought of as a *local procedure* (in the situation defined by the locally Lie groupoid (G, W)).

There is a *composition* due to Charles Ehresmann of these local procedures given by $s{*}t(x) = s(\beta t(x)) \circ t(x)$ where $\circ$ is the composition in the groupoid G. The domain of $s * t$ is usually smaller than that of t and may even be empty. Further the codomain of $s * t$ may not be a subset of W: thus the notion of smoothness of $s * t$ may not make sense. In other words, the composition of local procedures may not be a local procedure. Nonetheless, the set $\Gamma^\omega(\mathsf{G}, W)$ of all compositions of local procedures with its composition $*$ has the structure of an *inverse semigroup*, and it is from this that the Holonomy Groupoid, $\mathbf{Hol}(\mathsf{G}, W)$ is constructed as a Lie groupoid in Aof-Brown (1992), following details given personally by J. Pradines to Brown in 1981.

The motivation for this construction, due to Pradines, was to construct the *monodromy groupoid* $M(G)$ of a Lie groupoid G. The details are given in Brown and Mucuk (1994). The monodromy groupoid has this name because of the *monodromy principle* on the extendability of local morphisms. It is a *local-to-global* construction. It has a kind of *left adjoint* property given in detail in Brown and Mucuk (1994). So it has certain properties that are analogous to a van Kampen theorem.

The holonomy construction is applied to give a Lie structure to $M(G)$. When G is the pair groupoid $X \times X$ of a manifold X, then $M(G)$ is the *fundamental groupoid* $\pi_1 X$. It is crucial that this construction of $M(X)$ is independent of paths in X, but is defined by a suitable neighbourhood of the diagonal in $X \times X$, which is in the spirit of synthetic differential geometry, and so has the possibility of being applicable in wider situations. What is *unknown* is how to extend this construction to define *higher homotopy groupoids* with useful properties.

In a real quantum system, a *unique* holonomy groupoid may represent *parallel transport* processes and the *'phase-memorizing'* properties of such remarkable quantum systems. This theme could be then further pursued by employing *locally Lie groupoids in local-to-global procedures* (cf. Aof and Brown, 1993) for the construction in Quantum Spacetime of the *Holonomy Groupoid* (which is *unique*, according to the Globalization Theorem).

An alternative approach might involve the application of the more recently found fundamental theorems of Algebraic Topology –such as the Higher Homotopy generalization of the van Kampen theorem– to charactrize the *topological invariants* of a higher-dimensional topological space, for example in the context of AQFT, in terms of *known invariants* of its simpler subspaces. We also mention here the work of Brown and Janelidze (1997) which extends the van Kampen theorem to a purely *categorical* construction.

The *generalized* notion of a van Kampen Theorem, giving as it does ways in which an algebraic structure describing a large object is determined by the algebraic structures of of its pieces and the way they are glued, has many suggestive possibilities for both extensions and applications, and it should provide a basis for *higher dimensional, non-Abelian* methods in *local-to-global* questions in theoretical physics and Categorical Ontology, and therefore open up completely new fields.

1.8 The van Kampen Theorem and Its Generalizations to Groupoids and Higher Homotopy

The van Kampen Theorem 2.1 has an important and also anomalous rôle in algebraic topology. It allows computation of an important invariant for spaces built up out of simpler ones. It is anomalous because it deals with a nonabelian invariant, and has not been seen as having higher dimensional analogues.

However Brown, 1967, found a generalisation of this theorem to groupoids, as follows. In this, $\pi_1(X, X_0)$ is the fundamental *groupoid* of X on a set X_0 of base points: so it consists of homotopy classes rel end points of paths in X joining points of $X_0 \cap X$.

Theorem 1.3 (The Van Kampen Theorem for the Fundamental Groupoid, (Brown,1967)). *Let the space X be the union of open sets U, V with intersection W, and let X_0 be a subset of X meeting each path component of U, V, W. Then*
(C) (connectivity) *X_0 meets each path component of X, and*
(I) (isomorphism) *the diagram of groupoid morphisms induced by inclusions:*

$$\begin{array}{ccc} \pi_1(W, X_0) & \xrightarrow{\ i\ } & \pi_1(U, X_0) \\ {\scriptstyle j}\big\downarrow & & \big\downarrow{\scriptstyle k} \\ \pi_1(V, X_0) & \xrightarrow[\ l\ ]{} & \pi_1(X, X_0) \end{array} \qquad (1.15)$$

is a pushout of groupoids.

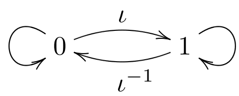
Theorem 2.1 discussed in Section 2 is the special case when $X_0 = \{x_o\}$. From Theorem 1.3 one can compute a particular fundamental group $\pi_1(X, x_0)$ using combinatorial information on the graph of intersections of path components of U, V, W. For this it is useful to develop some combinatorial groupoid theory, as in Brown, 2006, and Higgins, 1971. Notice two special features of this method:
(i) The computation of the *invariant* one wants to obtain, *the fundamental group*, is obtained from the computation of a larger structure, and so part of the work is to give methods for computing the smaller structure from the larger one. This usually involves non canonical choices, such as that of a maximal tree in a connected graph.
(ii) The fact that the computation can be done is surprising in two ways: (a) The fundamental group is computed *precisely*, even though the information for it uses input in two dimensions, namely 0 and 1. This is contrary to the experience in homological algebra and algebraic topology, where the interaction of several dimensions involves exact sequences or spectral sequences, which give information only up to extension, and (b) the result is a *non commutative invariant*, which is usually even more difficult to compute precisely. Thus exact sequences by themselves cannot show that a group is given as an HNN-extension: however such a description may be obtained from a pushout of groupoids, generalising the pushout of groupoids in diagram (see Brown, 2006).
The reason for this success seems to be that the fundamental groupoid $\pi_1(X, X_0)$ contains information in *dimensions 0 and 1*, and therefore it can adequately reflect the geometry of the intersections of the path components of U, V, W and the morphisms induced by the inclusions of W in U and V. This fact also suggested the question of whether such methods could be extended successfully to *higher dimensions.*
The following special case shows how the groupoid van Kampen Theorem gives an analogy between geometry and algebra. Let X be the circle S^1; choose U, V to be slightly extended semicircles including $X_0 = \{+1, -1\}$. Then $W = U \cap V$ is not path connected and so it is not clear where to choose a single base point. The day is saved by hedging one's bets, and using the two base points $\{+1, -1\}$. Now a key feature of groupoid theory is the groupoid I, the indiscrete groupoid on two objects $0, 1$, which acts as a unit interval object in the

category of groupoids. It also plays a rôle analogous to that of the infinite cyclic group Ç in the category of groups. One then compares the pushout diagrams, the first in spaces, the second in groupoids.

$$\begin{array}{ccc} \{0,1\} & \longrightarrow & \{0\} \\ \downarrow & & \downarrow \\ [0,1] & \longrightarrow & \mathbb{S}^1 \end{array} \qquad \begin{array}{ccc} \{0,1\} & \longrightarrow & \{0\} \\ \downarrow & & \downarrow \\ \mathsf{I} & \longrightarrow & \mathbb{Z} \end{array}$$

spaces groupoids

The left hand diagram shows the circle as obtained from the unit interval $[0, 1]$ by identifying, in the category of spaces, the two end points $0, 1$. The right hand diagram shows the infinite group of integers $\mathbb{Z}$ as obtained from the finite groupoid I, again by identifying $0, 1$, but this time in the category of groupoids. Thus groupoid theory satisfactorily models this geometry. The groupoid I with its special arrow $\iota : 0 \to 1$ has also the following property: if g is an arrow of a groupoid G then there is a unique morphism $\hat{g} : \mathsf{I} \to G$ whose value on ι is g. Thus the groupoid I with ι plays for groupoids the same role as does for groups the infinite cyclic group $\mathbb{Z}$ with the element 1: they are each free on one generator in their respective category. However we can draw a complete diagram of the elements of I as follows:



whereas we cannot draw a complete picture of the elements of $\mathbb{Z}$.
The fundamental group is a kind of anomaly in algebraic topology because of its *non-Abelian* nature. Topologists in the early part of the 20th century were aware that:

(1) The non–commutativity of the fundamental group was useful in applications; for path connected X there was an isomorphism

$$H_1(X) \cong \pi_1(X, x)^{\mathrm{ab}}.$$

(2) The abelian homology groups existed in all dimensions.

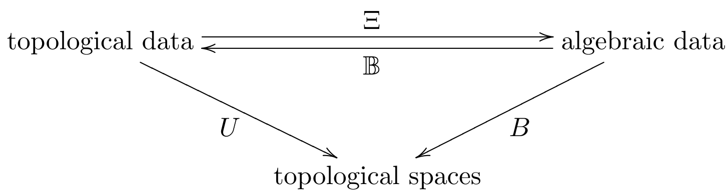
Consequently there was a desire to generalize the non-abelian fundamental group to all dimensions.

1.8.1 The Generalized Van Kampen Theorem (GvKT) for Covering Spaces and Covering Groupoids

There is yet another approach to the Van Kampen Theorem which goes *via* the theory of *covering spaces*, and the *equivalence* between covering spaces of a reasonable space X and functors $\pi_1(X) \longrightarrow \mathsf{Set}$ (Brown, 2005). See also an example (Douady and Douady, 1979) that consists in an exposition of the relation of this approach with the Galois theory. Another paper (Brown and Janelidze, 1997) gives a general formulation of conditions for the theorem to hold in the case $X_0 = X$ in terms of the map $U \sqcup V \to X$ being an *'effective global descent morphism'* (the theorem is given in the generality of lextensive categories). The latter work has been developed for topoi (Bunge and Lack, 2003). However, analogous interpretations of the topos work for higher dimensional Van Kampen theorems are not known so far.

The justification for changing from groups to groupoids is here threefold:
• the elegance and power of the results obtained with groupoids;
• the increased linking with other uses of groupoids (Brown, 2004), and
• the opening out of new possibilities in higher dimensions, which allowed for new results, calculations in homotopy theory, and also suggested new algebraic constructions.

The notion of the fundamental groupoid of a space goes back at least to Reidemeister (1934), and an exposition of the main theorems of 1-dimensional homotopy theory in terms of the fundamental groupoid $\pi_1(X, A)$ on a set A of base points was given by the first author in 1968, 1988 (see Brown et al. 2007). This was inspired by work of Philip Higgins in applying groupoids to group theory, (Higgins, 1966). The success of the applications to 1-dimensional homotopy theory, as perceived by the writer, led to the idea of using groupoids in higher homotopy theory, as announced in Brown (1967). There was an idea of a proof in search of a theorem. The chief obstacle was constructing and applying *higher homotopy groupoids*. The overall aim became subsumed in the following diagram:



(1.16)

The aim is to find suitable categories of topological data, algebraic data and functors as above, where U is the forgetful functor and $B = U \circ \mathbb{B}$, with the following properties:

(1) the functor Ξ is defined homotopically and satisfies a Higher Homotopy Seifert–van Kampen theorem (HHSvKT), in that it preserves certain colimits;

(2) $\Xi \circ \mathbb{B}$ is naturally equivalent to 1;

(3) there is a natural transformation $1 \longrightarrow \mathbb{B} \circ \Xi$ preserving some homotopical information.

The purpose of (1) is to allow some calculation of Ξ. This condition also rules out at present some widely used algebraic data, such as simplicial groups or groupoids, since for those cases no such functor Ξ is known. (2) shows that the algebraic data faithfully captures some of the topological data. The imprecise (3) gives further information on the algebraic modelling. The functor B should be called a *classifying space functor* because it often generalises the classifying space of a group or groupoid.

We explain more about the HHvKT, in the case when the topological data is that of a filtered topological space

$$X_* : \quad X_0 \subseteq X_1 \subseteq \cdots \subseteq X_n \subseteq \cdots \subseteq X. \tag{1.17}$$

The advantage of this situation is to hope to obtain global information on X by climbing up the 'ladder' of the subspaces X_n, which again may be considered 'local'. But now we consider 'local' in another sense by supposing that there is given a cover $\mathcal{U} = \{U^\lambda\}_{\lambda\in\Lambda}$ of X such that the interiors of the sets of $\mathcal{U}$ cover X. For each $\zeta \in \Lambda^n$ we set $U^\zeta = U^{\zeta_1} \cap \cdots \cap U^{\zeta_n}, U^\zeta_i = U^\zeta \cap X_i$. Then $U^\zeta_0 \subset U^\zeta_1 \subset \cdots$ is called the *induced filtration* U^ζ_* of U^ζ. Thus we can describe the filtered space X_* as a colimit in terms of the following diagram:

$$\bigsqcup_{\zeta\in\Lambda^2} U^\zeta_* \underset{b}{\overset{a}{\rightrightarrows}} \bigsqcup_{\lambda\in\Lambda} U^\lambda_* \xrightarrow{c} X_* \tag{1.18}$$

Here $\bigsqcup$ denotes disjoint union; a, b are determined by the inclusions $a_\zeta : U^\lambda \cap U^\mu \longrightarrow U^\lambda, b_\zeta : U^\lambda \cap U^\mu \longrightarrow U^\mu$ for each $\zeta = (\lambda, \mu) \in \Lambda^2$; and c is determined by the inclusions $c_\lambda : U^\lambda \longrightarrow X$. We would like this diagram to express that X_* is built from all the local filtered spaces U^λ_* by gluing them along the intersections $U^\zeta_* = U^\lambda_* \cup U^\mu_*$ whenever $\zeta = (\lambda, \mu)$. The useful categorical term for this is that diagram (1.18) is a *coequaliser diagram* in the category of filtered spaces.

We would like to turn this topological information into algebraic information. to enable us to understand and to calculate. So we apply the functor Ξ and if it preserves disjoint union we have the following diagram:

$$\bigsqcup_{\zeta\in\Lambda^2} \Xi(U^\zeta_*) \underset{b}{\overset{a}{\rightrightarrows}} \bigsqcup_{\lambda\in\Lambda} \Xi(U^\lambda_*) \xrightarrow{c} \Xi(X_*) \tag{1.19}$$

We would like this diagram (1.19) to be a coequaliser diagram in our category of algebraic data. This is not true in general but needs an extra condition, which we call *connected* for that topological data, not only on the U^λ_* but on all finite intersections of these. The conclusion of the HHvKT is then the important fact that X_* is also connected, and that diagram (1.19)is indeed a coequaliser diagram. This implies that the global algebraic invariant ΞX_* is *completely determined* by the local algebraic invariants ΞU^λ_*, and the way these are glued together using the information on the $\Xi\, U^\zeta_*$. Note that this is not a reductionist result: the whole is not just made up of its parts, but, as is only sensible, is made up of its parts and the way they are put together.

In the case the open cover consists of two elements, then the above coequaliser reduces to a pushout, and so includes the cases of the van Kampen Theorem considered earlier.

A feature of this scheme is that the algebraic data that we use has structure in a range of dimensions. This is necessary for homotopy theory since change in a low dimension can considerably affect higher dimensional behaviour. We do not define the connectivity condition precisely here, but note that while it does considerably restrict the range of applications, it still allows for new proofs of classical theorems of homotopy theory, such as the relative Hurewicz theorem,and allows allows for totally new results, including nonabelian results in dimension 2.

The format of the above coequaliser (1.19) is similar to diagrams appearing in Grothendieck's descent theory, but which extend to the left indefinitely. That theory is a very sophisticated local-to-global theory. This is perhaps indicative for future work.

The examples of *topological data* for which these schemes are known to work are:

topological data	algebraic data
spaces with base point	groups
spaces with a set of base points	groupoids
filtered spaces	crossed complexes
n-cubes of pointed spaces	cat^n-groups
Hausdorff spaces	double groupoids with connections

In fact crossed complexes are equivalent to a bewildering array of other structures, which are important for applications (Brown, 1999). Cat^n-groups are also equivalent to *crossed n-cubes of groups.* The construction of the equivalences and of the functors Ξ in all these cases is difficult conceptually and technically. The general philosophy is that one type of category is sufficiently geometric to allow for the formulation and proof of theorems, in a higher dimensional fashion, while another is more 'linear' and suitable for calculation. The transformations between the two forms give a kind of synaethesia. The classifying space constructions are also significant, and allow for information on the homotopy classification of maps.

From the ontological point of view, these results indicate that it is by no means obvious what algebraic data will be useful to obtain precise local-to-global results, and indeed new forms of this data may have to be constructed for specific situations. These results do not give a TOE, but do give a new way of obtaining new information not obtainable by other means, particularly when this information is in a non commutative form. The study of these types of results is not widespread, but will surely gain attention as their power becomes better known.

In Algebraic Topology crossed complexes have several *advantages* such as:

- They are good for *modelling CW-complexes.* Free crossed resolutions enable calculations with *small CW-models* of $K(G, 1)$s *and their maps* (Brown and Razak, 1999).
- Also, they have an interesting relation with the Moore complex of simplicial groups and of *simplicial groupoids.*
- They *generalise groupoids and crossed modules to all dimensions.* Moreover, the natural context for the second relative homotopy groups is crossed modules of groupoids, rather than groups.
- They are convenient for *calculation*, and the functor Π is classical, involving *relative homotopy groups.*
- They provide a kind of *'linear model'* for homotopy types which includes all 2-types. Thus, although they are not the most general model by any means (they do not contain quadratic information such as Whitehead products), this simplicity makes them easier to handle and to relate to classical tools. The new methods and results obtained for crossed complexes can be used as a model for more complicated situations. For example, this is how a general n-adic Hurewicz Theorem was found (Brown and Loday, 1987b)

- Crossed complexes have a *good homotopy theory*, with a *cylinder object, and homotopy colimits*. (A *homotopy classification* result generalises a classical theorem of Eilenberg-Mac Lane).

- They are close to chain complexes with a group(oid) of operators, and related to some classical homological algebra (e.g. *chains of syzygies*). In fact if SX is the simplicial singular complex of a space, with its skeletal filtration, then the crossed complex $\Pi(SX)$ can be considered as a slightly *non commutative version of the singular chains of a space.*

For more details on these points, we refer to Brown, 2004.

1.9 Construction of the Homotopy Double Groupoid of a Hausdorff Space

In the previous section, we mentioned that higher homotopy groupoids have been constructed for filtered spaces and for n-cubes of spaces. It is also possible to construct a homotopy double groupoid for a Hausdorff space, and prove a higher homotopy van Kampen theorem for this functor. This illustrates the interest and difficulty of extending this construction to other situations, such as smooth manifolds, or for Quantum Axiomatics.
We shall begin by recalling the construction of *The Homotopy Double Groupoid* $\boldsymbol{\rho}^{\square}(X)$ as adapted from Brown, Hardie, Kamps and Porter (2002), and the reader should refer to that source for complete details.

1.10 The singular cubical set of a topological space

We shall be concerned with the low dimensional part (up to dimension 3) of the singular cubical set

$$R^{\square}(X) = (R^{\square}_n(X), \partial_i^-, \partial_i^+, \varepsilon_i)$$

of a topological space X. We recall the definition (cf. Brown and Hardie, 1976). For $n \geqslant 0$ let

$$R^{\square}_n(X) = \mathsf{Top}(I^n, X)$$

denote the set of *singular n–cubes* in X, i.e. continuous maps $I^n \longrightarrow X$, where $I = [0,1]$ is the unit interval of real numbers. We shall identify $R^{\square}_0(X)$ with the set of points of X. For $n = 1, 2, 3$ a singular n–cube will be called a *path*, resp. *square*, resp. *cube*, in X. The *face maps*

$$\partial_i^-, \partial_i^+ : R^{\square}_n(X) \longrightarrow R^{\square}_{n-1}(X) \quad (i = 1, \ldots, n)$$

are given by inserting 0 resp. 1 at the i^{th} coordinate whereas the *degeneracy maps*

$$\varepsilon_i : R^{\square}_{n-1}(x) \longrightarrow R^{\square}_n(X) \quad (i = 1, \ldots, n)$$

are given by omitting the i^{th} coordinate. The face and degeneracy maps satisfy the usual cubical relations (cf Brown and Higgins (1981); Kamps and Porter (1997), § 1.1; § 5.1). A path $a \in R^{\square}_1(X)$ has *initial point* $a(0)$ and *endpoint* $a(1)$. We will use the notation

$a : a(0) \simeq a(1)$. If a, b are paths such that $a(1) = b(0)$, then we denote by $a + b : a(0) \simeq b(1)$ their *concatenation*, i.e.

$$(a+b)(s) = \begin{cases} a(2s) & 0 \leqslant s \leqslant \frac{1}{2} \\ b(2s-1) & \frac{1}{2} \leqslant s \leqslant 1 \end{cases}$$

If x is a point of X, then $\varepsilon_1(x) \in R_1^{\square}(X)$, denoted e_x, is the *constant path* at x, i.e.

$$e_x(s) = x \text{ for all } s \in I.$$

If $a : x \simeq y$ is a path in X, we denote by $-a : y \simeq x$ the *path reverse* to a, i.e. $(-a)(s) = a(1-s)$ for $s \in I$. In the set of squares $R_2^{\square}(X)$ we have two partial compositions $+_1$ (*vertical composition*) and $+_2$ (*horizontal composition*) given by concatenation in the first resp. second variable. Similarly, in the set of cubes $R_3^{\square}(X)$ we have three partial compositions $+_1, +_2, +_3$.

The standard properties of vertical and horizontal composition of squares are listed in Brown and Hardie (1976) §1. In particular we have the following *interchange law*. Let $u, u', w, w' \in R_2^{\square}(X)$ be squares, then

$$(u +_2 w) +_1 (u' +_2 w') = (u +_1 u') +_2 (w +_1 w')$$

whenever both sides are defined. More generally, we have an interchange law for rectangular decomposition of squares. In more detail, for positive integers m, n let $\varphi_{m,n} : I^2 \longrightarrow [0, m] \times [0, n]$ be the homeomorphism $(s, t) \longmapsto (ms, nt)$. An $m \times n$ *subdivision* of a square $u : I^2 \longrightarrow X$ is a factorization $u = u', \varphi_{m,n}$; its *parts* are the squares $u_{ij} : I^2 \longrightarrow X$ defined by

$$u_{ij}(s, t) = u'(s + i - 1, t + j - 1) \ .$$

We then say that u is the *composite* of the array of squares (u_{ij}), and we use matrix notation $u = [u_{ij}]$. Note that as in §1, $u +_1 u'$, $u +_2 w$ and the two sides of the interchange law can be written respectively as

$$\begin{bmatrix} u \\ u' \end{bmatrix}, \qquad [u \ \ w], \qquad [u \quad w \ u' \quad w']$$

Finally, *connections*:

$$\Gamma^-, \Gamma^+ : R_1^{\square}(X) \longrightarrow R_2^{\square}(X)$$

are defined as follows. If $a \in R_1^{\square}(X)$ is a path, $a : x \simeq y$, then let

$$\Gamma^-(a)(s, t) = a(\max(s, t));\ \Gamma^+(a)(s, t) = a(\min(s, t)).$$

The full structure of $R^{\square}(X)$ as a *cubical complex with connections and compositions* has been exhibited in (Al-Agl, Brown and Steiner, 2002).

1.10.1 Thin squares

In the setting of a geometrically defined double groupoid with connection, as in Brown and Hardy (1976), (resp. Brown, Hardie, Kamps and Porter, 2002), there is an appropriate notion of *geometrically thin* square. It is proved in Brown and Hardy (1976) as Theorem 5.2 (resp. Brown, Hardie, Kamps and Porter, 2002, Proposition 4), that in the cases given there, geometrically and algebraically thin squares coincide. In our context the explicit definition is as follows:

Definition 1.3. A square $u : I^2 \longrightarrow X$ in a topological space X is *thin* if there is a factorisation of u:

$$u : I^2 \xrightarrow{\Phi_u} J_u \xrightarrow{p_u} X,$$

where J_u is a tree and Φ_u is piecewise linear (PWL, see below) on the boundary ∂I^2 of I^2.

Here, by a *tree*, we mean the underlying space $|K|$ of a finite 1-connected 1-dimensional simplicial complex K.

A map $\Phi : |K| \longrightarrow |L|$ where K and L are (finite) simplicial complexes is PWL (*piecewise linear*) if there exist subdivisions of K and L relative to which Φ is simplicial.

Let u be as above, then the homotopy class of u relative to the boundary ∂I^2 of I is called a *double track*. A double track is *thin* if it has a thin representative.

1.11 The Homotopy Double Groupoid of a Hausdorff space

The full data for the homotopy double groupoid, $\boldsymbol{\rho}^{\square}(X)$, will be denoted by

$$(\boldsymbol{\rho}_2^{\square}(X), \boldsymbol{\rho}_1^{\square}(X), \partial_1^-, \partial_1^+, +_1, \varepsilon_1),\ (\boldsymbol{\rho}_2^{\square}(X), \boldsymbol{\rho}_1^{\square}(X), \partial_2^-, \partial_2^+, +_2, \varepsilon_2)$$
$$(\boldsymbol{\rho}_1^{\square}(X), X, \partial^-, \partial^+, +, \varepsilon).$$

Here $\boldsymbol{\rho}_1(X)$ denotes the *path groupoid* of X. We recall the definition. The objects of $\boldsymbol{\rho}_1(X)$ are the points of X. The morphisms of $\boldsymbol{\rho}_1^{\square}(X)$ are the equivalence classes of paths in X with respect to the following relation $\sim_T$.

Definition 1.4. Let $a, a' : x \simeq y$ be paths in X. Then a is *thinly equivalent* to a', denoted $a \sim_T a'$, if there is a thin relative homotopy between a and a'.

We note that $\sim_T$ is an equivalence relation, see Brown, Hardie, Kamps and Porter (2002). We use $\langle a \rangle : x \simeq y$ to denote the $\sim_T$ class of a path $a : x \simeq y$ and call $\langle a \rangle$ the *semitrack* of a. The groupoid structure of $\boldsymbol{\rho}_1^{\square}(X)$ is induced by concatenation, $+$, of paths. Here one makes use of the fact that if $a : x \simeq x',\ a' : x' \simeq x'',\ a'' : x'' \simeq x'''$ are paths then there are canonical thin relative homotopies

$$(a + a') + a'' \simeq a + (a' + a'') : x \simeq x''' \ (\textit{rescale})$$
$$a + e_{x'} \simeq a : x \simeq x';\ e_x + a \simeq a : x \simeq x' \ (\textit{dilation})$$
$$a + (-a) \simeq e_x : x \simeq x \ (\textit{cancellation}).$$

The source and target maps of $\boldsymbol{\rho}_1^{\square}(X)$ are given by

$$\partial_1^- \langle a \rangle = x, \;\; \partial_1^+ \langle a \rangle = y,$$

if $\langle a \rangle : x \simeq y$ is a semitrack. Identities and inverses are given by

$$\varepsilon(x) = \langle e_x \rangle \quad \text{resp.} - \langle a \rangle = \langle -a \rangle.$$

In order to construct $\boldsymbol{\rho}_2^{\square}(X)$, we define a relation of cubically thin homotopy on the set $R_2^{\square}(X)$ of squares.

Let u, u' be squares in X with common vertices. (1) A *cubically thin homotopy* $U : u \equiv_T^{\square} u'$ between u and u' is a cube $U \in R_3^{\square}(X)$ such that
(i) U is a homotopy between u and u',

$$\text{i.e. } \partial_1^-(U) = u, \;\; \partial_1^+(U) = u',$$

(ii) U is rel. vertices of I^2,

$$\text{i.e. } \partial_2^-\partial_2^-(U), \;\; \partial_2^-\partial_2^+(U), \;\; \partial_2^+\partial_2^-(U), \;\; \partial_2^+\partial_2^+(U) \text{ are constant,}$$

(iii) the faces $\partial_i^{\alpha}(U)$ are thin for $\alpha = \pm 1, \; i = 1, 2$.
(2) The square u is *cubically* T*-equivalent* to u', denoted $u \equiv_T^{\square} u'$ if there is a cubically thin homotopy between u and u'.

The relation $\equiv_T^{\square}$ can be seen to be an equivalence relation on $R_2^{\square}(X)$. For the proof of this result, the reader is referred to (Brown, Hardie, Kamps and Porter, 2002).
If $u \in R_2^{\square}(X)$ we write $\{u\}_T^{\square}$, or simply $\{u\}_T$, for the equivalence class of u with respect to $\equiv_T^{\square}$. We denote the set of equivalence classes $R_2^{\square}(X) \equiv_T^{\square}$ by $\boldsymbol{\rho}_2^{\square}(X)$. This inherits the operations and the geometrically defined connections from $R_2^{\square}(X)$ and so becomes a double groupoid with connections. A proof of the final fine detail of the structure is given in (Brown, Hardie, Kamps and Porter, 2002).
An element of $\boldsymbol{\rho}_2^{\square}(X)$ is *thin* if it has a thin representative (in the sense of Definition in Brown (2004a). From the remark at the beginning of this subsection we infer:

Lemma 1.2. *Let* $f : \boldsymbol{\rho}^{\square}(X) \to \mathsf{D}$ *be a morphism of double groupoids with connection. If* $\alpha \in \boldsymbol{\rho}_2^{\square}(X)$ *is thin, then* $f(\alpha)$ *is thin.*

Lemma 1.3. ***The Homotopy Addition Lemma.*** *Let* $u : I^3 \to X$ *be a singular cube in a Hausdorff space* X*. Then by restricting* u *to the faces of* I^3 *and taking the corresponding elements in* $\boldsymbol{\rho}_2^{\square}(X)$*, we obtain a cube in* $\boldsymbol{\rho}^{\square}(X)$ *which is commutative by the homotopy addition lemma for* $\boldsymbol{\rho}^{\square}(X)$ *(Brown, Hardie, Kamps and Porter, 2002, Proposition 5.5). Consequently, if* $f : \boldsymbol{\rho}^{\square}(X) \to \mathsf{D}$ *is a morphism of double groupoids with connections, then any singular cube in* X *determines a commutative 3-shell in* D*.*

Now under the situation given earlier where the Hausdorff space X has an cover by sets $\{U_\lambda\}_{\lambda \in \Lambda}$ we get a diagram as follows:

$$\bigsqcup_{\zeta\in\Lambda^2} \boldsymbol{\rho}^{\square}(U^{\zeta}) \underset{b}{\overset{a}{\rightrightarrows}} \bigsqcup_{\lambda\in\Lambda} \boldsymbol{\rho}^{\square}(U^{\lambda}) \xrightarrow{c} \boldsymbol{\rho}^{\square}(X) \tag{1.20}$$

The following is a statement of the Higher Homotopy van Kampen Theorem (HHvKT) expressed in terms of Double Groupoids with connections as developed and proven in (Brown, Hardie, Kamps and Porter, 2002).

Theorem 1.4 (Brown et al, 2004a.). ***The van Kampen theorem for Double Groupoids*** *If the interiors of the sets of $\mathcal{U}$ cover X, then in the above diagram* (1.20)*, c is the coequaliser of a, b in the category of double groupoids with connections.*

The reader is referred to [72] for the proof of this form of the Higher Homotopy van Kampen theorem.

A special case of this result is when $\mathcal{U}$ has two elements. In this case the coequaliser reduces to a pushout.
An important feature of the proof is the notion of commutative cube, the relation of these to thin cubes, and the fact that any multiple composition of commutative cubes is commutative. All these are facts whose analogues for squares are trivial. Thus the step from dimension to, i.e. for squares, to dimension 3, i.e. for cubes, is a large one technically and conceptually. Corresponding results in higher dimensions involve increasing difficulties, which are overcome for the groupoid case in [68] , and also in the category case in [142]–[143].

1.12 The Basic Principle of Quantization

At the microscopic/indeterministic level certain physical quantities assume only discrete values. The means of quantization describes the passage from a classical to an associated quantum theory where, at the probabilistic level, Bayesian rules are replaced by theorems on the composition of amplitudes. The classical situation is considered as 'commutative': one considers a pair (A, Π) where typically A is a commutative algebra of a class of continuous functions on some topological space and Π is a state on A. Quantization involves the transference to a 'non-commutative' situation via an integral transform: $(\mathsf{A}, \Pi) \longrightarrow (\mathcal{A}^{\mathrm{ad}}, \psi)$ where $\mathcal{A}^{\mathrm{ad}}$ denotes the self-adjoint part of the non-commutative Banach algebra $\mathcal{A} = \mathcal{L}(H)$, the bounded linear operators (observables) on a Hilbert space H. In this case, the state ψ can be specified as $\psi(T) = \mathrm{Tr}(\rho \mathrm{T})$, for T in $\mathcal{L}(H)$ and where ρ is a density operator. Alternative structures may involve a Poisson manifold (with Hamiltonian) and $(\mathcal{A}^{\mathrm{ad}}, \psi)$ possibly with time evolution. Such quantization procedures are realized by the transforms of Weyl-Heisenberg, Berezin, Wigner-Weyl-Moyal, along with certain variants of these. Problematic can be the requirements that the adopted quantum theory should converge to the clasical limlit, as $\hbar \longrightarrow 0$, meaning that in the Planck limit, $\hbar$ is small in relationship to other relevant quantities of the same dimension [164].

1.13 Quantum Effects

Let $\mathcal{H}$ be a (complex) Hilbert space (with inner product denoted $\langle\ ,\ \rangle$) and $\mathcal{L}(\mathcal{H})$ the bounded linear operators on on $\mathcal{H}$. We place a natural *partial ordering* "$\leqslant$" on $\mathcal{L}(\mathcal{H})$ by $S \leqslant T$ if

$$\langle S\psi, \psi\rangle \leqslant \langle T\psi, \psi\rangle \ , \ \text{for all } \psi \in \mathcal{H} \ .$$

In the terminology of Gudder [255], an operator $A \in \mathcal{H}$ is said to represent a *quantum effect* if $0 \leqslant A \leqslant I$. Let $\mathcal{E}(\mathcal{H})$ denote the set of quantum effects on $\mathcal{H}$. Next, let

$$P(\mathcal{H}) = \{P \in \mathcal{L}(\mathcal{H}) : P^2 = P \ , \ P = P^*\} \ ,$$

denote the space of projection operators on $\mathcal{H}$. The space $P(\mathcal{H}) \subseteq \mathcal{E}(\mathcal{H})$ constitutes the *sharp quantum effects* on $\mathcal{H}$. Likewise a natural partial ordering "$\leqslant$" can be placed on $P(\mathcal{H})$ by defining $P \leqslant Q$ if $PQ = P$.

A *quantum state* is specified in terms of a probability measure $m : P(\mathcal{H}) \longrightarrow [0, 1]$, where $m(I) = 1$ and if P_i are mutually orthogonal, then $m(\sum P_i) = \sum m(P_i)$. The corresponding quantum probabilities and stochastic processes, may be either "sharp" or "fuzzy". A brief mathematical formulation following Gudder (2004) accounts for these distinctions as will be explained next.

Let $\mathcal{A}(\mathcal{H})$ be a σ–algebra generated by open sets and consider the *pure states* as denoted by $\Omega(\mathcal{H}) = \{\omega \in \mathcal{H} : \|\omega\| = 1\}$. We have then relative to the latter an *effects space* $\mathcal{E}(\Omega(\mathcal{H}), \mathcal{A}(\mathcal{H}))$ less "sharp" than the space of projections $P(\mathcal{H})$ and thus comprising an entity which is *"fuzzy"* in nature. For a given *unitary operator* $U : \mathcal{H} \longrightarrow \mathcal{H}$, a *sharp observable* X_U is expressed abstractly by a map

$$X_U : \mathcal{A}(\mathcal{H}) \longrightarrow \mathcal{E}(\Omega(\mathcal{H}), \mathcal{A}(\mathcal{H})) \ ,$$

for which $X_U(A) = I_{U^{-1}(A)}$.
Suppose then we have a *dynamical group* ($t \in \mathbb{R}$) satisfying $U(s + t) = U(s)U(t)$, such as in the case $U(t) = \exp(-itH)$ where H denotes the energy operator of Schrödinger's equation. Such a group of operators extends X_U as above to a *fuzzy (quantum) stochastic process*

$$\widetilde{X}_{U(t)} : \mathcal{A}(\mathcal{H}) \longrightarrow \mathcal{E}(\Omega(\mathcal{H}), \mathcal{A}(\mathcal{H})) \ .$$

One can thus define classes of *analogous* quantum processes with 'similar' dynamic behaviour (see also our discussion in the previous *Section 7*) by employing dynamical group *isomorphisms*, whereas comparisons between dissimilar quantum processes could be represented by dynamical group *homomorphisms*.

1.14 Measurement Theories

1.14.1 Measurements and Phase–Space

We have already mentioned the issue of quantum measurement and now we offer a sketch of the background to its origins and where it may lead. Firstly, the question of measurement in quantum mechanics (QM) and quantum field theory (QFT) has flourished for about 75 years. The intellectual stakes have been dramatically high, and the problem rattled the development of 20th (and 21st) century physics at the foundations. Up to 1955, Bohr's Copenhagen school dominated the terms and practice of quantum mechanics having reached (partially) eye–to–eye with Heisenberg on empirical grounds, although not the case with Einstein who was firmly opposed on grounds on incompleteness with respect to physical reality. Even to the present day, the hard philosophy of this school is respected throughout most of theoretical physics. On the other hand, post 1955, the measurement problem adopted a new lease of life when von Neumann's beautifully formulated QM in the mathematically rigorous context of Hilbert spaces of states. As Birkhoff and von Neumann (1936) remark:

> "There is one concept which quantum theory shares alike with classical mechanics and classical electrodynamics. This is the concept of a mathematical "phase–space". According to this concept, any physical system $\mathfrak{C}$ is at each instant hypothetically associated with a "point" in a fixed phase–space Σ; this point is supposed to represent mathematically, the "state" of $\mathfrak{C}$, and the "state" of $\mathfrak{C}$ is supposed to be ascertained by "maximal" observations."

In this respect, *pure states* are considered as maximal amounts of information about the system, such as in standard representations using *position–momenta* coordinates [88].

The concept of 'measurement' has been argued to involve the influence of the Schrödinger equation for time evolution of the wave function ψ, so leading to the notion of entanglement of states and the indeterministic reduction of the wave packet. Once ψ is determined it is possible to compute the probability of measurable outcomes, at the same time modifying ψ relative to the probabilities of outcomes and observations eventually causes its collapse. The well–known paradox of Schrödinger's cat and the Einstein–Podolsky–Rosen (EPR) experiment are questions mooted once dependence on reduction of the wave packet is jettisoned, but then other interesting paradoxes have shown their faces. Consequently, QM opened the door to other interpretations such as 'the hidden variables' and the Everett–Wheeler assigned measurement within different worlds, theories not without their respective shortcomings. In recent years some countenance has been shown towards Cramer's 'advanced–retarded waves' transactional formulation in 1980, cited in [78]–[79], where $\psi\psi^*$ corresponds to a probability that a wave transaction has been finalized ('the quantum handshake').

Let us now turn to another facet of quantum measurement. Note firstly that QFT pure states resist description in terms of field configurations since the former are not always physically interpretable. Algebraic quantum field theory (AQFT) as expounded by Roberts in [228] points to various questions raised by considering theories of (unbounded) operator –valued distributions and nets of von Neumann algebras. Using in part a gauge theoretic approach, the idea is to regard two field theories as equivalent when their associated nets of observables are isomorphic. More specifically, AQFT considers taking (additive) nets of field algebras $\mathcal{O} \longrightarrow \mathcal{F}(\mathcal{O})$ over subsets of Minkowski space, which among other properties, enjoy Bose–Fermi commutation relations. Although at first glances there may be analogues with sheaf theory, these analogues are severely limited. The typical net does not give rise to a presheaf because the relevant morphisms are in reverse. Closer then is to regard a net as a precosheaf, but then the additivity does not allow proceeding to a cosheaf structure. This may reflect upon some incompatibility of AQFT with those aspects of quantum gravity (QG) where for example sheaf–theoretic/topos approaches are advocated (as for example in refs. [78]–[79]).

1.15 The Kochen-Specker (KS) Theorem

Arm–in–arm with the measurement problem goes a problem of 'the right logic', for quantum mechanical/complex biological systems and quantum gravity. It is well–known that classical Boolean truth–valued logics are patently inadequate for quantum theory. Logical theories founded on projections and self–adjoint operators on Hilbert space H do run into certain problems. One 'no–go' theorem is that of *Kochen–Specker* (KS) which for $\dim H > 2$, does not permit an evaluation (global) on a Boolean system of 'truth values'. Butterfield and Isham in [79], considered self–adjoint operators on H with purely discrete spectrum. The KS theorem was then interpreted as saying that a particular presheaf does not admit a global section. Partial valuations corresponding to local sections of this presheaf are introduced, and then generalized evaluations are defined. The latter enjoy the structure of a Heyting algebra and so comprise an intuitionistic logic. Truth values are describable in terms of sieve–valued maps, and the *generalized evaluations* are identified as *subobjects in a topos.* The further relationship with interval valuations motivates associating to the presheaf a von Neumann algebra where the supports of states on the algebra determines this relationship.

The above considerations lead directly to the next subsections which proceeds from linking quantum measurements with *Quantum Logics*, and then to the *construction* of spacetime structures on the basis of Quantum Algebra/Algebraic Quantum Field Theory (AQFT) concepts ; such constructions of QST representations as those presented in Sections 4 and 5 of Baianu et al. [38],[69] are based on the existing QA, AQFT and Algebraic Topology concepts, as well as several new QAT concepts that are being developed in this paper. For the QSS detailed properties, and also the rigorous proofs of such properties, the reader is referred to the recent book by Alfsen and Schultz [3]. We utilized in Sections 6 and 7 of ref.[38]. a significant amount of recently developed results in Algebraic Topology (AT), such as for example, the *Higher Homotopy van Kampen theorem* (see the relevant subsection in the Appendix for further mathematical details) to illustrate how constructions of QSS and QST, *non-Abelian* representations can be either generalized or extended on the basis of *GvKT*. We also employ the categorical form of the *CW–complex Approximation* (CWA) theorem) in Section 7 to both systematically construct such generalized representations of quantum space–time and provide, together with GvKT, the principal methods for determining the general form of the fundamental *algebraic invariants* of their *local or global*, topological structures. The algebraic invariant of Quantum Loop (such as, the graviton) Topology in QST is defined in Section 5 as the *Quantum Fundamental Groupoid (QFG)* of QST which can be then calculated– at least in principle – with the help of AT fundamental theorems, such as *GvKT*, especially for the relevant case of spacetime representations in *non–commutative* algebraic topology.

Several competing, tentative but promising, frameworks were recently proposed in terms of categories and the 'standard' topos for Quantum, Classical and Relativistic observation processes. These represent important steps towards developing a Unified Theory of Quantum Gravity, especially in the context-dependent measurement approach to Quantum Gravity [78]–[79]. The possibility of an unified theory of measurement was suggested in these reports in the context of both classical, Newtonian systems and quantum gravity. From their standpoint, Isham and Butterfield [78] proposed to utilize the concept of 'standard' topos [177] for further development of an unified measurement theory and quantum gravity (see also ref.

[79] for the broader aspects of this approach). Previous and current approaches to quantum gravity in terms of categories and higher dimensional algebra (especially, 2-categories) should also be mentioned in this context ([38] and references cited therein). Furthermore, time -as in Minkowski 'spacetime'- is not included in this mathematical concept of "most general space" and, therefore, from the beginning such quantum gravity theories appear to be heavily skewed in favor of the quantum aspects, at the expense of time as considered in the space-time of general relativity theory.
The first choice of logic in such a general framework for quantum gravity and context-dependent measurement theories was intuitionistic related to the set-theoretic and presheaf constructions utilized for a context-dependent valuation theory [78]. The attraction, of course, comes from the fact that a topos is arguably a very general, mathematical model of a 'generalized space' that involves an intuitionistic logic algebra in the form of a special distributive lattice called a *Heyting Logic Algebra*, as was discussed earlier.

1.16 Quantum Logics (QL) and Algebraic Logic (AL)

As pointed out by Birkhoff and von Neumann in 1930, a logical foundation of quantum mechanics consistent with quantum algebra is essential for both the completeness and mathematical validity of the theory [52]. With the exception of a non-commutative geometry approach to unified quantum field theories [85], the Isham and Butterfield framework in terms of the 'standard' Topos [177] and the 2-category approach by John Baez, cited in [38]; other quantum algebra and topological approaches are ultimately based on set-theoretical concepts and differentiable spaces (manifolds). Since it has been shown that standard set theory which is subject to the axiom of choice relies on Boolean logic (Diaconescu (1976), cited by Mac Lane and Moerdijk in ref.[177]), there appears to exist a basic logical inconsistency between the quantum logic–which is not Boolean–and the Boolean logic underlying all differentiable manifold approaches that rely on continuous spaces of points, or certain specialized sets of elements. A possible solution to such inconsistencies is the definition of a generalized Topos concept, and more specifically, of a Quantum Topos concept which is consistent with both Quantum Logic and Quantum Algebras, being thus suitable as a framework for unifying quantum field theories and physical modelling of complex systems in complex systems biology (CSB).
The problem of logical consistency between the quantum algebra and the Heyting logic algebra as a candidate for quantum logic is here discussed next. The development of Quantum Mechanics from its very beginnings both inspired and required the consideration of specialized logics compatible with a new theory of measurements for microphysical systems. Such a specialized logic was initially formulated by von Neumann and Birkhoff [52], and called 'Quantum Logic'. Subsequent research on Quantum Logics [88] resulted in several approaches that involve several types of non-distributive lattice (algebra) for n–valued quantum logics. Thus, modifications of the Łukasiewicz Logic Algebras that were introduced in the context of algebraic categories by Georgescu and Vraciu in 1970 [118], can provide an appropriate framework for representing quantum systems, or– in their unmodified form- for describing the activities of complex networks in categories of Łukasiewicz Logic Algebras [18].

1.16.1 Lattices and Von Neumann-Birkhoff (VNB) Quantum Logic: Definitions and Logical properties

Recall that an *s–lattice* **L**, or a 'set-based' lattice, was defined as a *partially ordered set* that has all binary products (defined by the *s*–lattice operation " $\bigwedge$") and coproducts (defined by the *s*–lattice operation "$\bigvee$ "), with the "partial ordering" between two elements X and Y belonging to the *s*–lattice being written as "$X \preceq Y$". The partial order defined by $\preceq$ holds in **L** as $X \preceq Y$ if and only if $X = X \bigwedge Y$ (or equivalently, $Y = X \bigvee Y$ Eq.(3.1)(p. 49 in [177]).

Categorical Definition of a Lattice

Utilizing the category theory concepts defined above, we need introduce a categorical definition of the concept of lattice that need be *'set–free'* in order to maintain logical consistency with the algebraic foundation of Quantum Logics and relativistic spacetime geometry. Such category–theoretical concepts unavoidably appear also in several sections of this paper as they provide the tools for deriving very important, general results that link Quantum Logics and Classical (Boolean) Logic, as well as pave the way towards a universal theory applicable also to semi-classical, or mixed, systems. Furthermore, such concepts are indeed applicable to measurements in complex biological networks, as it will be shown in considerable detail in a subsequent paper in this volume [40].

A *lattice* is defined as a category (see, for example refs. [165]–[166], [12] and [32]) subject to all ETAC axioms, (but not subject, in general, to the Axiom of Choice usually encountered with sets relying on (distributive) Boolean Logic), that has all binary products and all binary coproducts, as well as the following 'partial ordering' properties:

(i) when unique arrows $X \longrightarrow Y$ exist between objects X and Y in **L** such arrows will be labelled by "$\preceq$ ", as in " $X \preceq Y$";

(ii) *the coproduct* of X and Y , written as "$X \bigvee Y$" will be called the "*sup object*, or "*the least upper bound*", whereas the product of X and Y will be written as "$X \bigwedge Y$", and it will be called an *inf object*, or "*the greatest lower bound*";

(iii) *the partial order* defined by $\preceq$ holds in **L** , as $X \preceq Y$ if and only if $X = X \bigwedge Y$ (or equivalently, $Y = X \bigvee Y$ (p. 49 in [177]).

If a lattice **L** has **0** and **1** as objects, such that $0 \longrightarrow X \longrightarrow 1$ (or equivalently, such that $0 \preceq X \preceq 1$) for all objects X in the lattice **L** viewed as a category, then **0** and **1** are the unique, initial, and respectively, terminal objects of this concrete category **L**. Therefore, **L** has all finite limits and all finite colimits (p. 49 in [177]), and is said to be *finitely complete and co-complete.*

Alternatively, the lattice 'operations' can be defined via functors in a 2-category (for definitions of functors and 2-categories see, for example, refs. [176], [71] and Section 9 of Baianu et al. in [38]), as follows:

$$\bigwedge : L \times L \longrightarrow L \ , \qquad \bigvee : L \times L \to L \qquad (1.21)$$

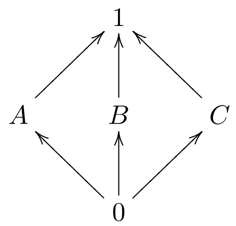
and $0, 1 : 1 \to L$ as a "lattice object" in a 2-category with finite products.
A lattice is called *distributive* if the following identity :

$$X \bigwedge (Y \bigvee Z) = (X \bigwedge Y) \bigvee (X \bigwedge Z) \ . \tag{1.22}$$

holds for all X, Y, and Z objects in **L**. Such an identity also implies the dual distributive lattice law:

$$X \bigvee (Y \bigwedge Z) = (X \bigvee Y) \bigwedge (X \bigvee Z) \ . \tag{1.23}$$

(Note how the lattice operators are 'distributed' symmetrically around each other when they appear in front of a parenthesis.) A *non-distributive* lattice is not subject to either restriction (13.13) or (13.14). An example of a non-distributive lattice is:



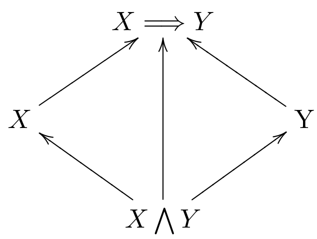
(1.24)

1.16.2 Definitions of an Intuitionistic Logic Lattice

A *Heyting algebra*, or *Brouwerian lattice*, H, is a *distributive lattice* with all finite products and coproducts, and which is also *cartesian closed*. Equivalently, a Heyting algebra can be defined as a distributive lattice with both initial (0) and terminal (1) objects which has an "exponential" object defined for each pair of objects X, Y, written as: "$X \Rightarrow Y$" or Y^X, such that:

$$Z = (X \Rightarrow Y) \iff Z = X^Y \ , \tag{1.25}$$

In the Heyting algebra, $X \Rightarrow Y$ is a least upper bound for all objects Z that satisfy the condition $Z = X^Y$. Thus, in terms of a categorical diagram, the partial order in a Heyting algebra can be represented as



(1.26)

A lattice will be called complete when it has all small limits and small colimits (e.g., small products and coproducts, respectively). It can be shown (p.51 in [177]) that any complete and infinitely distributive lattice is a Heyting algebra.

1.17 Łukasiewicz Quantum Logic (LQL)

With all assertions of the type *system A* is excitable to the i-th level and system B is excitable to the j-th level" on e can form a distributive lattice, **L** (as defined above). The composition laws for the lattice will be denoted by $\bigcup$ and $\bigcap$. The symbol $\bigcup$ will stand for the logical non-exclusive 'or', and $\bigcap$ will stand for the logical conjunction 'and'. Another symbol "$\preceq$ " allows for the ordering of the levels and is defined as *the canonical ordering* of the lattice. Then, one is able to give a symbolic characterization of the system dynamics with respect to each energy level i. This is achieved by means of the maps $\delta t : L \rightarrow L$ and $N : L \rightarrow L$, (with N being the negation). The necessary logical restrictions on the actions of these maps lead to an *n-valued Łukasiewicz Algebra*:

(I) There is a map $N : L \longrightarrow L$, so that

$$N(N(X)) = X \ , \tag{1.27}$$

$$N(X \bigcup Y) = N(X) \bigcap N(Y) \tag{1.28}$$

and

$$N(X \bigcap Y) = N(X) \bigcup N(Y) \ , \tag{1.29}$$

for any $X, Y \in \mathbf{L}$.

(II) There are $(n-1)$ maps $\delta i : L \longrightarrow L$ which have the following properties:

(a) $\delta i(0) = 0,\ \delta i(1) = 1$, for any $1 \leqslant i \leqslant n-1$;

(b) $\delta i(X \bigcup Y) = \delta i(X) \bigcup \delta i(Y), \delta i(X \bigcap Y) = \delta i(X) \bigcap \delta i(Y)$,
for any $X, Y \in \mathbf{L}$, and $1 \leqslant i \leqslant n-1$;

(c) $\delta i(X) \bigcup N(\delta i(X)) = 1, \delta i(X) \bigcap N(\delta i(X)) = 0$, for any $X \in \mathbf{L}$;

(d) $\delta i(X) \subset \delta 2(X) \subset ... \subset \delta(n-1)(X)$, for any $X \in \mathbf{L}$;

(e) $\delta i * \delta j = \delta i$ for any $1 \leqslant i, j \leqslant n-1$;

(f) If $\delta i(X) = \delta i(Y)$ for any $1 \leqslant i \leqslant n-1$, then $X = Y$;

(g) $\delta i(N(X)) = N(\delta j(X))$, for $i + j = n$.

(as defined by Georgescu and Vraciu in 1970 [118]).

The first axiom states that the double negation has no effect on any assertion concerning any level, and that a simple negation changes the disjunction into conjunction and conversely. The second axiom presents ten sub–cases that are summarized in equations (a) - (g). Sub–case (IIa) states that the dynamics of the system is such that it maintains the structural integrity of the system. It does not allow for structural changes that would alter the lowest and the highest energy levels of the system. Thus, maps $\delta : L \longrightarrow L$ are chosen to represent the dynamic behaviour of the quantum or classical systems in the absence of structural changes. Equation (IIb) shows that the maps (d) maintain the type of conjunction and disjunction. Equations (IIc) are chosen to represent assertions of the following type: ⟨the sentence "a system component is excited to the i-th level or it is not excited to the same level" is true⟩, and ⟨the sentence "a system component is excited to the i-th level and it is not excited to the same level, at the same time" is always false⟩.

Equation (IId) actually defines the actions of maps δt. Thus, Eq. (I) is chosen to represent a change from a certain level to another level as low as possible, just above the zero level of **L**. $\delta 2$ carries a certain level x in assertion X just above the same level in $\delta 1(X)$, $\delta 3$ carries the level x-which is present in assertion X–just above the corresponding level in $\delta 2(X)$, and so on. Equation (IIe) gives the rule of composition for the maps δt. Equation (IIf) states that any two assertions that have equal images under all maps δt, are equal. Equation (IIg) states that the application of δ to the negation of proposition X leads to the negation of proposition $\delta(X)$, if $i + j = n$.

In order to have the n-valued Łukasiewicz Logic Algebra represent correctly the basic behaviour of quantum systems (observed through measurements that involve a quantum system interactions with a measuring instrument –which is a macroscopic object), several of these axioms have to be significantly changed so that the resulting lattice becomes non-distributive, possibly non-commutative, and also non–associative [88].

On the other hand for classical systems, modelling with the unmodified Łukasiewicz Logic Algebra can include both stochastic and fuzzy behaviour. For an example of such models the reader is referred to previous publications [18], [23] modelling the activities of complex genetic networks from a classical standpoint. One can also define as in [118] the 'centers' of certain types of Łukasiewicz n-Logic Algebras. In addition to the important Adjointness Theorem for such Centered Łukasiewicz n-Logic Algebras which actually defines an equivalence relation [118], one can formulate the following conjecture.

Conjecture 1.1. *There exist adjointness relationships, respectively, between each pair of the Centered Heyting Logic Algebra,* ***Bl****, and the Centered* ***CLuk****–n Categories.*

Remark 1.1. R1. Both a Boolean Logic Algebra and a Centered Łukasiewicz Logic Algebra can be represented as/are Heyting Logic algebras (the converse is, of course, generally false!).

R2. The natural equivalence logic classes defined by the adjointness relationships in the above Adjointness Theorem define a fundamental, *'logical groupoid'* structure.

Note also that the above Łukasiewicz Logic Algebra is *distributive* whereas the quantum logic requires a *non-distributive* lattice of quantum 'events'. Therefore, in order to generalize the standard Łukasiewicz Logic Algebra to the appropriate Quantum Łukasiewicz Logic Algebra, LQL, axiom I needs modifications, such as : $N(N(X)) = Y \neq X$ (instead of the restrictive identity $N(N(X)) = X$, and, in general, giving up its 'distributive' restrictions, such as

$$N(X \bigcup Y) = N(X) \bigcap N(Y) \text{ and } N(X \bigcap Y) = N(X) \bigcup N(Y)\ , \qquad (1.30)$$

for any X, Y in the Łukasiewicz Quantum Logic Algebra, LQL, whenever the context, 'reference frame for the measurements', or 'measurement preparation' interaction conditions for quantum systems are incompatible with the standard 'negation' operation N of the Łukasiewicz Logic Algebra that remains however valid for classical systems, such as various complex networks with n-states (cf. [18]).

1.18 Quantum Fields, General Relativity and Symmetries

As the experimental findings in high-energy physics–coupled with theoretical studies– have revealed the presence of new fields and symmetries, there appeared the need in modern physics to develop systematic procedures for generalizing space–time and Quantum State Space (QSS) representations in order to reflect these new concepts.

In the General Relativity (GR) formulation, the local structure of space–time, characterized by its various tensors (of energy–momentum, torsion, curvature, etc.), incorporates the gravitational fields surrounding various masses. In Einstein's own representation, the physical space–time of GR has the structure of a Riemannian R^4 space over large distances, although the detailed local structure of space–time – as Einstein perceived it – is likely to be significantly different.

On the other hand, there is a growing consensus in theoretical physics that a valid theory of Quantum Gravity requires a much deeper understanding of the small(est)–scale structure of Quantum Space–Time (QST) than currently developed. In Einstein's GR theory and his subsequent attempts at developing a unified field theory (as in the space concept advocated by Leibnitz), space-time does *not* have an *independent existence* from objects, matter or fields, but is instead an entity generated by the *continuous* transformations of fields. Hence, the continuous nature of space–time was adopted in GR and Einstein's subsequent field theoretical developments. Furthermore, the quantum, or 'quantized', versions of space-time, QST, are operationally defined through local quantum measurements in general reference frames that are prescribed by GR theory. Such a definition is therefore subject to the postulates of both GR theory and the axioms of Local Quantum Physics. We must emphasize, however, that this is *not* the usual definition of position and time observables in 'standard' QM. Therefore, the general reference frame positioning in QST is itself subject to the Heisenberg uncertainty principle, and therefore it acquires through quantum measurements, a certain 'fuzziness' at the Planck scale which is intrinsic to all microphysical quantum systems,

1.19 Applications of the Van Kampen Theorem to Crossed Complexes. Representations of Quantum Space-Time in terms of Quantum Crossed Complexes over a Quantum Groupoid.

There are several possible applications of the generalized van Kampen theorem in the development of physical representations of a quantized space-time 'geometry'. For example, a possible application of the generalized van Kampen theorem is the construction of the initial, quantized space-time as the *unique colimit* of *quantum causal sets (posets)* which was precisely described in ref.[69] in terms of *the nerve of an open covering $N\mathcal{U}$* of the topological space X that would be isomorphic to a k-simplex K underlying X. The corresponding,*noncommutative* algebra Ω associated with the finitary T_0-poset $P(S)$ is *the Rota* *algebra* Ω discussed in [69], and the *quantum topology* T_0 is defined by the partial ordering arrows for regions that can overlap, or superpose, coherently (in the quantum sense) with each other. When the poset $P(S)$ contains $2N$ points we write this as $P_{2N}(S)$. The *unique* (up to an isomorphism) $P(S)$ in the *colimit*, $\lim_{\leftarrow} P_N X$, recovers a space homeomorphic to X [69]. Other non-Abelian results derived from the generalized van Kampen theorem were discussed by Brown, Hardie, Kamps and Porter in [72], and by Brown, Higgins and Sivera in [68].

1.20 Local–to–Global (LG) Construction Principles consistent with Quantum 'Axiomatics'

A novel approach to QST construction in AQFT may involve the use of fundamental theorems of algebraic topology generalised from topological spaces to spaces with structure, such as a filtration, or as an n-cube of spaces. In this category are the generalized, *Higher Homotopy Seifert-van Kampen theorems (HHSvKT)* of Algebraic Topology with novel and unique non-Abelian applications. Such theorems have allowed some new calculations of homotopy types of topological spaces. They have also allowed new proofs and generalisations of the classical Relative Hurewicz Theorem by R. Brown and coworkers [68],[72]. One may find links of such results to the expected *'non-commutative* geometrical' structure of quantized space–time [85].