Particle Physics - Problem Sheet 5

Discussion Question

D1 The effective Higgs potential can be written as:

$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

and the vacuum Higgs field is written as a doublet:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

(a) Show that the Higgs potential has a minimum with $\phi^\dagger\phi \neq 0$ only if $\mu^2 > 0$ and $\lambda > 0$. (For the purposes of this discussion you should entertain the possibility that the constant μ^2 could be less than 0.)

(b) What is the vacuum expectation value v at the minimum?

(c) What is the value of the Higgs potential at the minimum?

(d) By considering an excitation of the field, $h(x)$, about the minimum:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

show that the Higgs potential acquires a term $\lambda v^2 h^2$ and explain why this can be thought of as coming from a physical Higgs boson with a mass:

$$m_H = \sqrt{2\lambda}\, v$$

(e)* What are the next two terms in the expansion? What do they represent?

Standard Questions

1. (a) Write down all the allowed final states of Z-boson decay.

   (b) The width of each of the decays is:

$$\Gamma(Z \to f\bar{f}) = \frac{g_W^2[(c_V^f)^2 + (c_A^f)^2]}{48\pi\cos^2\theta_W} m_Z = 322[(c_V^f)^2 + (c_A^f)^2][\text{MeV}]$$

Where $c_V^f = T_3 - 2Q\sin^2\theta_W$ is the vector coupling and $c_A^f = T_3$ is the axial-vector coupling. Calculate the total width of the Z-boson and the branching ratios to the following experimentally observed final states:

- Each flavour of charged lepton: $e^+e^-, \mu^+\mu^-, \tau^+\tau^-$.
- Hadrons
- Nothing! (What particles give no signature in the detector?)

The numerical values for c_V^f and c_A^f are given on slide 13 of lecture 17.

2. The Z^0 boson is the linear combination of the B^0 boson that couples to weak hypercharge ($Y = 2(Q - T_3)$) with a strength $g'_W/2$ and the W^3 boson that couples with weak isospin (T^3) with a strength g_W:

$$Z^0 = W^3 \cos\theta_W - B^0 \sin\theta_W$$

(Note that using $\cos\theta_W$ and $\sin\theta_W$ ensures that the Z^0 boson is properly normalised.)

(a) Write down the value of T_3, and hence Y for left-handed and right-handed fermions, f.

(b) Show that the coupling of the Z^0 boson to left-handed fermions is:

$$c_{\mathrm{L}} = g_Z(T_3 - Q\sin^2\theta_W)$$

and to the right handed fermions is:

$$c_{\mathrm{R}} = -g_Z Q \sin^2\theta_W$$

Where $g_Z \equiv g_W/\cos\theta_W$ and $(e =)g_W \sin\theta_W = g'_W \cos\theta_W$.

(c) Show the the vector (c_V) and axial-vector (c_A) coupling constants can be written as:

$$c_V = c_{\mathrm{L}} + c_{\mathrm{R}} \quad c_A = c_{\mathrm{R}} - c_{\mathrm{L}}$$

and hence that the couplings c_V and c_A are as given in question 1.

3. (a) What are the allowed decay modes of the W-boson?

(b) What the branching ratios for leptons ($W \to \ell\nu$) and hadrons ($W \to q\bar{q}$)?

At the LEP collider operating at a centre-of-mass energy $\sqrt{s} = 200$ GeV, pairs of W bosons are produced by the process $e^+e^- \to W^+W^-$. What are the relative fractions of the detected W^+W^- final states?

$$\ell^+\nu_\ell\ell^-\bar{\nu}_\ell \qquad q\bar{q}\ell^-\bar{\nu}_\ell \qquad q\bar{q}q\bar{q}$$

(Note that ℓ represents any lepton flavor, and q any quark flavor)

4. Draw a Feynman diagram describing Higgs production by gluon-gluon fusion at the LHC.

Describe the signatures that you would expect to see in the detectors at the LHC for the decays of:

- A light neutral Higgs boson $M_H = 120$ GeV
- A heavier neutral Higgs boson with $M_H \approx 2M_Z$
- (*) A supersymmetric charged Higgs boson H^+

For the decay $H^0 \to W^+W^-$ discuss which final states from the W decays are the most favourable for measuring the Higgs mass.