Particle Physics

Lecture 12: Hadron Decays

- Resonances
- Heavy Meson and Baryons
- Decays and Quantum numbers
- CKM matrix
From Friday: Mesons and Baryons Summary

- Quarks are confined to *colourless* bound states, collectively known as **hadrons**:
  - **mesons**: quark and anti-quark. Bosons (s=0, 1) with a symmetric colour wavefunction.
  - **baryons**: three quarks. Fermions (s=1/2, 3/2) with antisymmetric colour wavefunction.
  - **anti-baryons**: three anti-quarks.
- Lightest mesons & baryons described by isospin \( I, I_3 \), strangeness \( S \) and hypercharge \( Y \)
  - isospin \( I = \frac{1}{2} \) for \( u \) and \( d \) quarks; (isospin combined as for spin)
  - \( I_3 = +\frac{1}{2} \) (isospin up) for up quarks; \( I_3 = -\frac{1}{2} \) (isospin down) for down quarks
  - \( S = +1 \) for strange quarks (additive quantum number)
  - hypercharge \( Y = S + B \)
- Hadrons display SU(3) flavour symmetry between \( u \), \( d \) and \( s \) quarks. Used to predict the allowed meson and baryon states.
- As baryons are fermions, the overall wavefunction must be **anti-symmetric**. The wavefunction is product of colour, flavour, spin and spatial parts: \( \psi = \chi_c \chi_f \chi_S \chi_L \) an odd number of these must be anti-symmetric.
  - consequences: no \( uuu \), \( ddd \) or \( sss \) baryons with total spin \( J = \frac{1}{2} \) \( (S = \frac{1}{2}, L = 0) \)
  - Residual strong force interactions between colourless hadrons propagated by mesons.

Resonances

- Hadrons which decay due to the strong force have very short lifetime \( \tau \approx 10^{-24} \text{ s} \)
- Evidence for the existence of these states are **resonances** in the experimental data
- Shape is Breit-Wigner distribution:
  - \( \sigma = \sigma_{\text{max}} \frac{\Gamma^2}{\Gamma^2 + (E - M)^2} \)
  - (recall \( \Gamma \) is calculated using Fermi’s Golden rule)

![Resonance Graph](image)
Discovery of the Heavy Quarks

- Collider experiments discovered the charm (1974), bottom (1977) and top quarks (1995)
- Produced in pairs e.g. $e^+e^- \rightarrow c\bar{c}$, $e^+e^- \rightarrow b\bar{b}$, $p\bar{p} \rightarrow t\bar{t}$
- At threshold ($E \sim 2m_q$) bound $c\bar{c}$, $b\bar{b}$ states are narrow resonances

- Heavy quarks mainly produced in pairs at hadron colliders ($c\bar{c}$, $b\bar{b}$, $t\bar{t}$)
- Single $c, b, t$ production requires an intermediate $W$ boson
- Heavy quark jets can be identified by jet tagging as the lightest $c$- and $b$-hadrons have significant lifetimes $\tau_c \sim 0.4\text{ps}$, $\tau_b \sim 1.5\text{ps}$

**$t\bar{t} \rightarrow W^+b \ W^-\bar{b}$ candidate event**

- Lines are project paths of charged particles through the detector.
- Not all particles originate from collision point.
- Particle produced and travelled short distance before decaying, indicates production of a $b$-quark!
Heavy Quark Mesons and Baryons

- Can define an SU(4) symmetry $u \leftrightarrow d \leftrightarrow s \leftrightarrow c \leftrightarrow 
- Heavy quark mesons and baryons obtained by replacing one (or more) of the light $u,d,s$ quarks by a heavy $c$ or $b$ quark
- There are no bound state hadrons containing $t$ quarks
- Lowest lying charm meson states with $M(D) \sim 1.9$ GeV:
  $D^+ (c \bar{d}), D^- (\bar{c} d), D^0 (c \bar{u}), D_s^+ (c \bar{s}), D_s^- (\bar{c} s)$
- Lowest lying bottom meson states with $M(B) \sim 5.3$ GeV:
  $B^0 (b \bar{d}), B^0 (b \bar{u}), B^- (b \bar{u}), B^+ (b \bar{u}), B_s^0 (b \bar{s}), B_s^0 (b \bar{s})$
- Baryons $\Lambda_c (cud), \Lambda_b (bud)$ ...
- Charmonium ($c \bar{c}$ ) and Bottomonium ($b \bar{b}$)

Health warning: do not attempt to remember the quark flavour content of these states!
(Just $p = uud, n = duu, \pi^- = ud, \pi^0 = uu, dd$)

Charmonium & Bottomonium

- Analogous to hydrogen spectroscopy with pairs of quarks and quark-quark potential $V_{qq}(r) = -4/3 \alpha_S/r + kr$ (see lecture 9)
Decays of Charmonium

- The J/ψ meson is a c ¯c state. It must decay to particles without charm quarks as \( M(J/ψ) < 2 M(D) \).
- Two options: decay via three gluons (to conserve colour) or one photon.

\[ \begin{align*}
\pi^+ & \quad \pi^0 \\
\pi^- & \quad J/ψ \\
\pi^- & \quad J/ψ
\end{align*} \]

- Decay rate is suppressed by \( α_S^6(q_{\text{gluon}}) \), giving the electromagnetic decay a change.
- The J/ψ meson lives for a relatively long time, giving rise to narrow resonance in e.g. \( e^+e^- \rightarrow \text{hadrons} \).

\[ \begin{align*}
\bar{s}(c) & \quad \bar{s}(c) \\
\bar{s}(c) & \quad \bar{s}(c)
\end{align*} \]

- Similar phenomena occurs in decays of s ¯s and b ¯b mesons.

Decays of Hadrons

- The proton is the only completely stable hadron
- The free neutron has a weak decay (~15 mins)
- Decay length of a particle is distance it travels before decaying \( L=βγct \)

<table>
<thead>
<tr>
<th>Force</th>
<th>Typical ( τ ) (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>QCD</td>
<td>( 10^{-20} - 10^{-23} )</td>
</tr>
<tr>
<td>QED</td>
<td>( 10^{-20} - 10^{-16} )</td>
</tr>
<tr>
<td>Weak</td>
<td>( 10^{-13} - 10^{3} )</td>
</tr>
</tbody>
</table>

- \( π^+, K^+, K_L^0 \) mesons are long-lived (~10 ns) and have weak decays
  - These particles live long enough to travel outside radii of collider detectors (~10 m)
- \( K_S^0 \) mesons and \( Λ^0 \) hyperons are long-lived (~100 ps) and have weak decays with decay lengths of ~ cm.

- \( π^0 \rightarrow γγ \), \( η \rightarrow γγ \) are electromagnetic decays, reconstructed from pairs of photons.
  \[ m_{π}^2 = (p(γ_1) + p(γ_2))^2 \]
- \( ρ, ω, φ, K^*, Λ, Σ^*, Ξ^* \) are resonances with strong decays.
  - Reconstructed as broad structures with widths \( \sim 100 \text{ MeV} \).
Decay Conservation Laws

- Relevant quantum numbers are:
  - isospin \((I, I_3)\)
  - parity \((P)\)
  - quark flavour: described using strangeness \((S=N(s)\rightarrow N(s))\), charm \((C=N(c)\rightarrow N(c))\), bottomness \((B=N(b)\rightarrow N(b))\)
- Baryon number and lepton numbers are always conserved!

<table>
<thead>
<tr>
<th>Baryon number</th>
<th>Isospin, (I)</th>
<th>(I_3)</th>
<th>Flavour, (S, C, B)</th>
<th>Parity, (P)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>EM</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Weak</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
</tbody>
</table>

Charged Pion Decay

- See problem sheet 1
- \(\pi^+\) consists of \(u\bar{d}\), lightest charged meson
- \(\tau(\pi^+) = 26\) ns
  - must decay via weak force, weak is the only force that can violate quark flavour
- Use hadronic form factor \(f_\pi\sim m_\pi\) to account for finite size of pion

\[
\mathcal{M} = [\bar{v}(d)g_W V_{ud} f_\pi \gamma^\mu (1 - \gamma_5) u(u)] \frac{1}{q^2 - m_W^2} [\bar{u}(\nu_\mu) g_W \gamma^\mu (1 - \gamma_5) v(\mu^+)]
\]

\[
\approx V_{ud} f_\pi \frac{g_W^2}{m_W^2} [\bar{v}(d) \gamma^\mu (1 - \gamma_5) u(u)] [\bar{u}(\nu_\mu) \gamma^\mu (1 - \gamma_5) v(\mu^+)]
\]

\[
|\mathcal{M}|^2 = 4 G_F^2 |V_{ud}|^2 f_\pi^2 m_\mu^2 [p_\mu \cdot p_\nu]
\]

\[
\Gamma = \frac{|\mathcal{P}|^*}{8\pi m_1^2} |\mathcal{M}|^2 = \frac{G_F^2}{8\pi} |V_{ud}|^2 f_\pi^2 m_\mu^2 (m_\pi^2 - m_\mu^2)
\]
Charged Kaon Decays

• Charged kaon is $\bar{s}u$ with $m_K = 498$ MeV
• lightest mesons containing strange quarks ⇒ must decay by weak force
• $\tau(K^+) = 12$ ns

• Leptonic decays
  • $\text{BR}(K^+ \rightarrow \mu^+\nu_\mu) = 63\%$
  • Kaon decay constant, $f_K = 160$ MeV
  • $V_{us} = 0.22$

• Semileptonic decays
  • $\text{BR}(K^+ \rightarrow \pi^0\mu^+\nu_\mu) = 3.8\%$
  • $\text{BR}(K^+ \rightarrow \pi^0\nu_e\bar{e}) = 5.1\%$

• Hadronic Decays
  • $\text{BR}(K^+ \rightarrow \pi^0\pi^+) = 21\%$
  • $\text{BR}(K^+ \rightarrow \pi^0\pi^0\pi^0) = 5.6\%$

Cabibbo-Kobayashi-Maskawa Matrix

• **Mass eigenstates** and **weak eigenstates** of quarks are not identical.
  - Decay properties measure mass eigenstates with a definite lifetime and decay width
  - The weak force acts on the weak eigenstates.

• Weak eigenstates are admixture of mass eigenstates, conventionally described using CKM matrix a mixture of the down-type quarks:

\[
\begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix} =
\begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}
\]

• e.g. weak eigenstate of the strange quark is a mixture between down, strange and bottom mass eigenstates
  \[s' = V_{cd}d + V_{cs}s + V_{cb}b\]

• The CKM matrix is unitary, $V_{\text{CKM}}^\dagger V_{\text{CKM}} = 1$; standard parameterisation in terms of three mixing angles ($\theta_1$, $\theta_2$, $\theta_3$) and one phase ($\delta$) is:

\[
\begin{pmatrix}
  \cos \theta_1 & \sin \theta_1 \cos \theta_3 & \sin \theta_1 \sin \theta_3 \\
  -\sin \theta_1 \cos \theta_3 & \cos \theta_1 \cos \theta_2 \cos \theta_3 - \sin \theta_2 \sin \theta_3 e^{i\delta} & \cos \theta_1 \cos \theta_2 \sin \theta_3 + \sin \theta_2 \cos \theta_3 e^{i\delta} \\
  \sin \theta_1 \sin \theta_2 & -\cos \theta_1 \sin \theta_2 \cos \theta_3 - \cos \theta_2 \sin \theta_3 e^{i\delta} & -\cos \theta_1 \sin \theta_2 \sin \theta_3 + \cos \theta_2 \cos \theta_3 e^{i\delta}
\end{pmatrix}
\]
Nobel Prize in Physics 2008

Awarded to Makoto Kobayashi, High Energy Accelerator Research Organization (KEK), Tsukuba, Japan and Toshihide Maskawa, Yukawa Institute for Theoretical Physics (YITP), Kyoto University, and Kyoto Sangyo University, Japan

"for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature"

Summary: Decays of Hadrons

- Hadron decays give us insight into forces in particle physics.
- **Strong decays** are characterised by very short lifetimes, $\tau \sim 10^{-20} - 10^{-23}$ s appearing as resonances with a large width $\Gamma \sim \text{MeV}$.
  - Final states are all hadronic. All quantum numbers are conserved.
- **Electromagnetic decays** are characterised by $\tau \sim 10^{-20} - 10^{-16}$ s.
  - Decays containing photons are electromagnetic.
  - All quantum numbers conserved except total isospin, $I$.
- **Weak decays** characterised by long lifetimes, $\tau \sim 10^{-13} - 10^{-3}$ s.
  - Responsible for decay of lightest baryons with a strange, charm or bottom quark.
  - Particles can live long enough to reach the detector.
  - Final states may be leptonic, semi-leptonic or hadronic.
  - Allows access to the elements of the **CKM matrix**.
  - Isospin, $I, I_3$, Parity, $P$, Flavour quantum numbers not conserved.
- **CKM matrix** relates the mass eigenstates to the weak eigenstates. Contains a complex phase.
Experimental Measurements of CKM Matrix

- Many measurements made by the BaBar and Belle experiments.
- Both study $e^+e^-$→ $\Upsilon(4S)$ → $B^0 \bar{B}^0$ to measure the decays of $b$ and $c$ quarks, e.g. $V_{cb}$ and $V_{ub}$

![BaBar 2007 graph](Babar 2007 graph)

$B \rightarrow D^* \ell \nu$ decays (as function of $D^*$ recoil) measures $|V_{cb}| = 0.0374 \pm 0.0017$

![Exclusive $B \rightarrow \pi \ell \nu$](Exclusive $B \rightarrow \pi \ell \nu$ graph)

Exclusive $B \rightarrow \pi \ell \nu$ (as a function of $q^2$) $|V_{ub}| = 0.0034(5)$

The Wolfenstein Parameterisation

- An expansion of the CKM matrix in powers of $\lambda = V_{us} = 0.22$

$$V_{CKM} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & -A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

- Parameterisation reflects almost diagonal nature of CKM matrix:
  - The diagonal elements $V_{ud}, V_{cs}, V_{tb}$ are close to 1
  - Elements $V_{us}, V_{cd} \sim \lambda$ are equal
  - Elements $V_{cb}, V_{ts} \sim \lambda^2$ are equal
  - Elements $V_{ub}, V_{td} \sim \lambda^3$ are very small

- Diagonal structure means quark mass eigenstate is almost equal to down quark weak eigenstate
  - similarly for strange and bottom mass eigenstates
- Note that the complex phase $\eta$ only appears in the very small elements, and is thus hard to measure.